A Tale of Two Workers: The Macroeconomics of Automation*

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Abstract

Industrialized countries have experienced a significant drop in the fraction of the population employed in middle wage, "routine" task-intensive occupations. Applying machine learning techniques, we identify the types of individuals who used to be work in such occupations and track their labor market outcomes. Based on these findings, we develop a quantitative, heterogeneous agent, general equilibrium model of labor force participation, occupational choice, and capital investment to study the aggregate and distributional effects of advances in automation. We use this framework as a laboratory to evaluate various public policies aimed at addressing the disappearance of routine employment.

Keywords: Polarization, Automation, Routine Employment, Labor Force Participation, Universal Basic Income, Unemployment Insurance, Retraining.

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1. Introduction

Advances in "automation technologies" have left an indelible mark on the labor market in industrialized economies such as the U.S. This is especially true in the past 40 years. An important literature demonstrates that these economies have experienced a significant drop in the fraction of the population employed in jobs in the middle of the occupational wage distribution (see, for instance, Autor-Katz-Kearney, 2006, Goos-Manning, 2007, Goos-Manning-Salomons, 2009, Autor, 2010, Acemoglu-Autor, 2011). This hollowing out of the middle is linked to the decline of occupations focused on "routine" tasks—those activities that can be performed by following a well-defined set of instructions and procedures. The nature of these routine tasks make them prime candidates to be performed by new automation technologies (see Autor-Levy-Murnane, 2003, and the subsequent literature).

This paper contributes to our understanding of this phenomenon along three dimensions. First, on the empirical side, we apply machine learning techniques that allow us to track over almost four decades, the employment outcomes of the *types* of individuals who used to work in routine occupations. This enables us to study what has happened to workers with "routine occupational characteristics" who might otherwise be working in such jobs. Second, based on our empirical findings, we develop a quantitative heterogeneous agent general equilibrium model of labor force participation, occupational choice, and investment dynamics, in the face of advances in automation technologies which is consistent with our findings. Armed with this empirically relevant model, we study the aggregate and distributional effects of automation both in terms of welfare and allocations. Third, we use this new theoretical framework as a "laboratory" to evaluate various public policies. The policies we consider belong to two distinct sets; those aimed at counteracting the negative effects of automation technologies, and a broader set that address the challenges associated with employment disappearance. In what follows we discuss each of these three parts in detail.

In Section 2, we apply machine learning techniques to micro-level data from the BLS Current Population Survey (CPS hereafter). This allows us to classify individuals, in an agnostic and formal manner, into different categories based on "occupational likelihood" and to identify what "routine characteristics" are in practice. Armed with this algorithm, we track the evolution of individuals with such characteristics over time, and ask what has happened to the type of workers who used to be employed in routine occupations.

We address the following questions. Are "routine-type" workers employed in different occupations now than they used to be (e.g. what would have been the 1980 predicted occupation of a 50 year old "routine type" man in 2018)? Did routine-type workers drop out of the labor force (in which case our algorithm informs us as to what the occupation of workers outside the labor force would have been)? Do routine-type workers tend to be more unemployed than they used to be?

Our key finding is that about two-thirds of such individuals ended up as labor force non-participants; the remaining one third ended up in non-routine manual occupations, that tend to be located at the bottom of the occupational wage distribution. We view this as crucial in addressing the following two issues: (i) the welfare and allocation implications of automation, and (ii) the evaluation of various public policies. As such, these finding guide us in developing an empirically relevant, quantitative general equilibrium model to address these issues.

We briefly describe the structure of the model, with detailed presentation in Section 3. Given the central role of occupational employment to the analysis, we consider a model with three occupations: (i) Non-Routine Cognitive, (ii) Non-Routine Manual, and (iii) Routine, representing the high, low, and middle of the occupational wage distribution, respectively. Given our quantitative goal, we focus attention on a tangible measure of automation and its technological progress. Specifically, we use measures of Information-Communication-Technology capital (ICT hereafter) that have been shown to capture various aspects of automation (see Eden and Gaggl (2018)).[1] As such, our analysis does not attempt to capture other ways to model automation (e.g., the adoption of new modes of production, as in Kopytov et al., 2018) or broader forces (such as globalization or offshoring) resulting in the changing occupational structure of employment, but rather concentrate in the effects of a *measurable* specific channel.

These factors of production (employment in three distinct occupations, and service flow from ICT and non-ICT capital) serve as inputs in the production of a final good. Perfectly competitive final good producers own the two capital stocks and invest in them optimally; hence, the degree of ICT adoption is endogenous. This allows us to study the effect of changes in the environment of the economy and policies on the degree of automation adoption, which in turn, affects the various decision margins in the model.

With respect to labor, the model features two-dimensional heterogeneity in Routine and Non-Routine Manual ability. Based on their idiosyncratic abilities, and the equilibrium compensation in Routine and Non-Routine-Manual employment, workers optimally make two decisions. First, they decide whether or not to participate in the labor force. Second, conditional on participating, they sort themselves into occupations and are either employed or unemployed given the matching frictions. We consider a search-and-matching framework with free entry equilibrium with the key occupational labor markets; the matching process between unemployed workers and vacancies posted by entrepreneurs is subject to a search friction as is common in the Diamond-Mortensen-Pissarides model (DMP hereafter).

Finally, on the government side we require that all government policies (such as unemployment insurance

[1]However, since overall ICT capital accounts for only a small part of the overall capital stock we also model non-ICT capital.

and transfers to those outside of the labor force) are financed with labor income and profit taxation. This requirement holds for all the policy experiments we consider.

Following the presentation of the model, we characterize the equilibrium in Section 4. Section 5 includes a discussion of the model calibration and quantitative results; in particular, we consider predictions for key *non-targeted* empirical moments when advance in automation technology is attributed solely to the observed increase in ICT productivity. We find that across these moments the model accounts for between a third and half of the empirical moments. For example, the model accounts for about half of the fall in Routine employment observed in the data, and half of the fall in the labor share of aggregate income. In the model, as in the data (See Eden and Gaggl (2018)), the fall in aggregate labor share is solely driven by the fall in Routine occupations' share of aggregate income. Finally, the model accounts for about a third of the movement in the average relative wage of Routine Non-Routine Manual workers.

Given this, we proceed to use the model as a "laboratory" to evaluate the allocation and distributional effects of various policies. Importantly, the general equilibrium structure ensures that the government budget constraint holds under each policy proposal. In Section 6 we consider two sets of policies. In the first set, we study the effects of policies that are aimed directly at counteracting the effects of ICT on Routine employment. These include "ICT taxing" and "occupation retraining". The retraining policies are of particular interest since they consider the equilibrium consequences of policies that change the abilities of workers in the face of improvements in ICT.

In the second set of policies, we consider a broader set of policy proposals. These include, the introduction of Universal Basic Income (UBI hereafter), changes in unemployment insurance benefit, changes in the minimum wage, and changes in the transfers for those that are out of the labor force. Again, for each of these policies we analyze their aggregate and distributional effects, both in terms of welfare and allocations.

Finally, Section 7 concludes the paper, while the different Appendices discuss various robustness checks, both empirically and theoretically.

2. Employment and Occupation Trends

An important literature documents the changes in the task content of work, its relation to the decline in the cost of computing and the rise in automation, and its implications for the structure of occupational employment and wages (see for example Autor, Levy and Murnane (2003), Acemoglu and Autor (2011), David and Dorn (2013) and Atalay et al. (2018)). Here, we document what has happened to workers employed in (and those who would otherwise be employed in) declining routine occupations. Are these individuals employed in other occupations? Do they tend to be more frequently unemployed? Or are they more likely to be non-participants in the labor force? Answering these questions is crucial for understanding the welfare implications of automation, as well as in the evaluation of policies that are meant to support workers affected by automation.

We face three challenges in addressing these questions. First, given that the process of automation started decades ago, one would need very long panels to assign workers to pre-polarization occupations. Such long panels are either small (e.g. the Panel Study of Income Dynamics, PSID; see Cortes, 2016), focused on a very specific cohort of individuals, or both (e.g. the National Longitudinal Survey of Youth, NLSY). Second, a person's occupation is not observed when s/he is out of the labor force. This is not a data collection problem per se, but rather a fundamental feature of labor force non-participation. In principle, one could attach the occupation in which the person was most recently employed. But this is questionable for individuals undergoing a prolonged spell of non-participation, and conceptually nonviable for those without previous employment spells. Indeed, and potentially because of these considerations, even in rich cross-sectional datasets like the CPS, occupations of labor force non-participants persons are not recorded. Third, even for the employed, we observe the occupation that workers chose in the presence of automation. It does not reveal the occupational choice of individuals "who would otherwise have been working in a routine occupation" in the counterfactual scenario in which polarization had not occurred.

We address these issues as follows. We consider an empirical framework that classifies individuals according to their *likelihood* of employment in various occupational groups based on their observed characteristics during late 1980s. As such, we track the *type* of people who used to work in specific (e.g. routine task-intensive) occupations prior to (or, at least, during an early period of) automation. We then follow over time the employment and occupational choices of individuals classified by their "likely" occupations. This allows us to answer the question of where declining, routine occupational worker types "end up" with the rise in automation. This analysis is considered in section 2.1.

2.1. Where do workers in declining occupations end up? A machine learning approach

2.1.1. Empirical Methodology and Data

Here we classify individuals aged 25-64 from the CPS into types based on the occupation they would most likely have been employed in before the rise of automation. To obtain such a classification, we apply a random forest, machine learning (hereafter ML) algorithm using information on age, education, gender, and race in a flexible manner. The occupational categories draw on the job polarization literature, relying on the task intensity of the occupation along two key dimensions. The first delineation is whether an occupation is routine or non-routine. The second is based on whether it is “cognitive” versus “manual” in task intensity. We thus end up with four categories of occupations: non-routine-cognitive (NRC); routing-cognitive (RC); non-routine-manual (NRM); and routine-manual (RM). For our occupation classification we follow Jaimovich and Siu (2012). For more details about variable and sample definitions see Appendix 8.

We use cross-sectional data on employed and unemployed individuals, and their current or most recent occupation, during the pre-automation period (defined 1984-1989) to train the ML algorithm to associate occupations to observable individual-level characteristics.[2] We then apply the algorithm to classify persons to “likely” occupations in the remaining CPS subsamples. First, we use the predictions to assign the “most likely” occupation to labor force non-participants during the pre-automation period. Second, we roll the predictions forward, to cross-sections from 1990 to 2017, and predict occupations to the entire sample. In so doing, we assign for each person in the CPS data, her/his most likely occupation based on pre-automation characteristics. This allows us to predict participation and occupational choice had automation not occurred.

2.1.2. Results

Our ML approach classifies each person (at each point in time) into their “most likely” of the four occupational groups (NRC, RC, NRM, and RM). We present our main results aggregating to two workers types – NRC and non-NRC (i.e., RC, RM, and NRM) workers – both for expositional clarity and for substantive reasons. In particular, predictive power is high and classification errors are small at this level of aggregation, allowing for the minimization of noise in the worker type-specific series for employment and occupational choice.[3] Finally, as documented in Cortes (2016) and Cortes et al. (2016), there are large difference in observable characteristics between NRC and all other worker types; R (RC and RM) and NRM types are much more similar. This fact motivates previous labor market models (such as Autor-Katz-Kearney, 2006; and Cortes et al., 2017) as well as our modeling choice below focusing on two broad groups of workers.

For male non-NRC types, Table 1 displays the of the fraction of workers in – or *propensity* to choose – labor force non-participation, unemployment, and in employment in three occupation categories – namely, NRC, NRM and R. The first two columns of Table 1 summarize these findings. In the late-1980s the fraction of non-NRC types employed in routine occupations was about 0.67; by 2017 this had dropped to approximately 0.57, a 16 log point fall.

The decline in routine employment is necessarily accompanied by an offsetting increase in other labor market statuses. Where did these non-NRC type men end up in 2017? As indicated by Table 1, they did not

[2]We pick 1989 as the benchmark year for comparisons, since per capita Routine employment peaked that year.

[3]In Appendix 8 we discuss the classification errors over the two types of workers, and explain in detail how we can get rid of the noise in the series which is added by those errors.

go into high-wage NRC occupations; their propensity to work in NRC remained essentially constant at zero throughout the period. By contrast, the probability of non-participation in the labor force (NLF) increased dramatically from 0.17 to 0.24. The share employed in NRM occupations increased as well, from about 0.11 to 0.15. Hence, the changes in NRM and NLF probability can account for the entire fall in R employment – about two-thirds of the decline can be accounted by the increase in NLF, and the rest by the increase in NRM employment. This is a key result of our analysis – on average, non-NRC types leaving routine employment are worse off. Dropping out of the labor force is likely to be consistent with some dependency on welfare payments, while a transition to NRM is likely to be accompanied by a decline in earnings.[4]

Finally, the bottom two rows of Table 1 show that there is no apparent trend in either the (population) fraction of the non-NRC who are unemployed, or in their unemployment rate. The stability of unemployment implies that even though exit from the labor force seems to have increased for non-NRC type men, among participants the employed-to-unemployed ratio has remained roughly constant. Appendix Figure 4 explores this further. Using CPS data on unemployment and employment by occupation, we document unemployment rates by different occupations over time. Consistent with the results shown above for the non-NRC and NRC type persons, it demonstrates that within each occupation, both the unemployment rate and exit rates show cyclical patterns but no trends over time.

We now verify that the increase in NLF and NRM are a unique phenomena to non-NRC types, and that this is not an economy-wide phenomena. Columns 3 and 4 of Table 1 summarize the changes in labor force and occupational employment status for NRC type men. This group has seen a decrease in NRC employment propensity.[5] But there is very little decline in labor force participation, no change in employment at NRM occupations, and a slight increase in R employment.[6] Hence this suggests that the changes discussed above for the non-NRC are particularly linked to the decline of R occupations.

While Table 1 display labor market changes for non-NRC men, similar patterns can be observed for non-NRC women but over a different time period. As is well known, the last 40 years of the twentieth century was characterized by a pronounced increase in labor force participation among women. But since the turn of the twenty-first century, female labor force participation has plateaued and begun to fall, even among the prime-aged. As such, our view is that the period since the turn of the twenty-fist century is more indicative of the female occupation dynamics.

Columns 1 and 2 of Table 2 present the same information as 1 but for female non-NRC types, during the period of 2001-2017. As is obvious, these female workers have seen a decline in their likelihood of em-

[4]See, for instance, David and Dorn (2013)

[5]See, Cortes et al. (2018) for analysis of the differing gender trends of men and women in the high-skilled labor market.

[6]See, Beaudry et al. (2018) for a model with "crowding in" of high-skilled workers into middle-paying R occupations.

Table 1: Labor market status and occupation composition changes for men, 1989-2017 by type

	non-NRC		NRC	
	(1)	(2)	(3)	(4)
	1989	2017	1989	2017
Population Weight	0.65	0.52	0.35	0.48
Fraction in R	0.67	0.57	0.02	0.06
Fraction in NRM	0.11	0.15	~0	0.01
Fraction in NRC	0.01	~0	0.99	0.90
Fraction in NLF	0.17	0.24	~0	0.03
Fraction in Unemployment	0.05	0.04	~0	0.01
Unemployment rate	0.06	0.06	~0	0.01

Notes:

ployment in R occupations. There have been no obvious increases in the propensity toward unemployment or working in NRC as a consequence. Instead, non-NRC women have seen offsetting increases in both the likelihood of non-participation and employment in NRM occupations; this split is approximately two-thirds toward NLF, one-third toward NRM. This is the same split that has been observed within non-NRC men over the the longer 1989-2017 time horizon, and, as Columns 3 and 4 of 2 show, also during the same period of 2001-2017. Again, their fall in R employment was offset by an increase in non-participation and NRM employment. And again, this split is approximately two-thirds, one-third. [7]

Hence, to summarize, when considering the behavior of non-NRC men either over the long horizon of 1989-2017 or the shorter horizon os 2001-2017 we find that the fall in R employment was offset by an increase in non-participation and NRM employment. And again, this split is approximately two-thirds, one-third. Similarly, when considering the behavior of of non-NRC women over the period following the trend increase in labor force participation (i.e. 2001-2017) we find a similar split. As such, we consider these labor market changes to be critical facts to match in our quantitative model analysis.

3. Model

Here, we present a heterogeneous agent model with occupational choice. Motivated by the findings of Table 1, indicating a sharp distinction between NRC and non-NRC characteristic types we consider two agent

[7] As with the NRC men, these dynamics are not observed within NRC women.

Table 2: Labor market status and occupation composition changes for non-NRC types

	female		male	
	(1)	(2)	(3)	(4)
	2001	2017	2001	2017
Population Weight	0.68	0.55	0.58	0.52
Fraction in R	0.39	0.30	0.64	0.57
Fraction in NRM	0.17	0.21	0.12	0.15
Fraction in NRC	0.07	0.06	0.01	~0
Fraction in NLF	0.34	0.40	0.19	0.24
Fraction in Unemployment	0.03	0.03	0.04	0.04
Unemployment rate	0.05	0.06	0.05	0.06

Notes:

types, referred to as high-skill (NRC) and low-skill (non-NRC) for simplicity.

High skill workers are identical and supply labor inelastically as employment in the non-routine cognitive (NRC) occupation. Low-skill agents choose whether to participate in the labor market, and if they do, whether to seek employment in the routine (R) or non-routine manual (NRM) occupation. The occupational labor markets for low-skill workers are subject to a search and matching friction as in Diamond (1982), Mortensen (1982) and Pissarides (1985). For tractability, there is full information about worker abilities and matching markets are fully segmented: unemployed workers and vacancies meet in occupation-*and-ability*-specific matches. The presence of frictions means unemployment is well-defined (as opposed simply to an employed versus non-employed distinction in frictionless models). We formally model unemployment as a labor market state since, in Section 6, some of the policy experiments we consider affect allocations, income distribution, and welfare through the incentive effects on job search and vacancy creation.

In addition to the three occupational labor inputs, capital input in the forms of ICT capital and non-ICT capital are used in final good production. Both capital stocks are owned by perfectly competitive final good producers who make investment decisions. Hence, the degree of automation in the form ICT capital accumulation is endogenous (see Eden and Gaggl (2018) who document the role of ICT for routinization and the dynamics of labor income share).

For technical reasons and tractability, we assume that the high-skilled are "capitalists" and own all firm equity in the economy; hence, low-skilled workers are excluded from asset/credit markets and are "hand to mouth," with current consumption equal to current income. This assumption regarding asset ownership, while simplified, has traction the data. For example, the Survey of Consumer Finances (SCF) reports median households net worth by education of household head. There we observe that over the period 1989-2016 median net worth of college graduates are more than 12 times as large as high school dropouts, and more than 4 times as large as high school graduates. Thus, highly educated individuals, who are empirically more likely to be NRC workers, own the vast majority of assets in the US.

The low-skilled are heterogeneous, and each worker is endowed with two ability parameters (productivity draws), one for occupation R and one for occupation NRM. Given their abilities in each occupation, individuals make two decisions: whether to participate in the labor force or not, and conditional on participation, in which occupation to search for employment. These choices depend on job finding probabilities and the equilibrium compensation when employed.

Finally, to allow for analysis of various government policies, we include the following taxes and transfers: a proportional tax on firms' profits, a proportional progressive tax on labor income, unemployment benefits, transfers to labor force non-participants, and (potentially) unconditional lump sum transfers.

3.1. Final Good Producers

Perfectly competitive, final good firms produce output (Y) using five inputs: NRC, R, and NRM intermediate goods (or service flows), denoted Y_{NRC}, Y_{NRM}, and Y_R, respectively; ICT capital (X_A); and non-ICT capital such as structures (K), which we refer to as simply "physical capital". The constant returns to scale production function for the final good is:

$$Y_t = Z_t K_t^{\gamma} \left((1-\eta) \left[(1-\alpha) Y_{NRC,t}^{EOS1} + \alpha \left[X_A^{\nu} + Y_{R,t}^{\nu} \right]^{\frac{EOS1}{\nu}} \right]^{\frac{EOS2}{EOS1}} + \eta Y_{NRM,t}^{EOS2} \right)^{\frac{1-\gamma}{EOS2}} \tag{1}$$

where Z_t denotes Hicks-neutral productivity, ν which controls the elasticity of substitution between ICT capital and the R intermediate good, EOS_1 which controls the elasticity of substitution between the NRC intermediate good and the ICT-R composite, EOS_2 which controls the elasticity of substitution between NRM and the composite of the previously discussed factors, and η and α which control the income shares of different factors of production.

Final good producers accumulate physical and ICT capital stocks (which depreciate at rates δ_K and δ_A, respectively) and purchase the three intermediate goods from competitive markets at the prevailing prices.[8] The relative price of investment in non-ICT is denoted $\phi_{K,t}$; the relative price of ICT capital is denoted $\phi_{A,t}$.

Hence, the firm's per-period profit is:

$$\pi = Y - P_R Y_R - P_{NRM} Y_{NRM} - P_{NRC} Y_{NRC} - \phi_A \left(X_A' - (1-\delta_A) X_A \right) - \phi_K \left(K' - (1-\delta_K) K \right)$$

[8] The model is isomorphic if we assume that the final good firm also rents the capital from intermediate capital services producers.

where we specify the final good to be the numeraire ($P_Y = 1$) and denote the prices of the intermediate goods by P_R, P_{NRC}, P_{NRM}. The firm's dynamic problem is:

$$V(K, X_A, \Lambda) = \max_{K', X'_A, Y_R, Y_{NRM}, Y_{NRC}} \left\{ (1 - T_\pi)\, \pi + \mathbb{E}\left[\Theta V\left(K', X'_A, \Lambda'\right)\right] \right\}$$

where T_π is a tax rate on firms' profits, $\mathbb{E}$ is the expectation operator, Θ is the stochastic discount factor, and $\Lambda = \{\phi_K, \phi_A, Z, T_\pi, Y_R, Y_{NRM}, Y_{NRC}\}$ is a vector that contains all the state variables that the representative firm takes as given, which are either exogenously specified or determined in equilibrium.[9]

The firm chooses the optimal accumulation of physical and ICT capital in accordance with two (standard) Euler equations that equalize current cost and future returns:

$$\phi_K = \mathbb{E}\left[\Theta \times \left(MPK' + (1-\delta)\, \phi'_K\right]\right. \tag{2}$$

$$\phi_A = \mathbb{E}\left[\Theta \times \left(MPA' + (1-\delta)\, \phi'_A\right]\right. \tag{3}$$

where MPK and MPA denote the marginal products of the two types of capital. Note that because firm profits are taxed net of investment costs costs, there are no equilibrium effects on optimal capital demand.[10]

3.2. Productivity, Intermediate Goods Production, and Labor Demand

An exogenously specified fraction of workers are high-skill, and supply their labor inelastically in a frictionless labor market toward producing the NRC good. They receive a market wage equal to their marginal revenue product. Every high-skill worker produces $y_{NRC} = f_{NRC}$ units of the NRC intermediate goods. Each low-skill agent draws a pair of idiosyncratic productivity parameters ε_R and ε_{NRM} from a joint distribution $\Gamma(\varepsilon_R, \varepsilon_{NRM})$; ε_R denotes the idiosyncratic ability of the worker if employed in production of the R intermediate good, and ε_{NRM} denotes the ability in the NRM good. A worker who is employed in R production with ability ε_R produces $y_R = f_R \times \varepsilon_R$ units of the R good; similarly, a worker employed in NRM with ε_{NRM} produces $y_{NRM} = f_{NRM} \times \varepsilon_{NRM}$. The productivity of intermediate goods production, f_R, f_{NRM}, f_{NRC} are exogenous.

[9] In writing the firms problem this way we already impose consistency conditions such that the optimal choice is identical across firms and therefore represents the aggregate. As we show below, prices of intermediate goods are determined by the optimal demand and therefore by aggregate quantities of the intermediate goods.

[10] See for a similar approach Abel (2007)

The labor markets for the low-skilled are frictional and fully segmented by good i and ability ε_i, for $i = \{R, NRM\}$. Unemployed low-skill workers choose whether to search in the R or NRM market or whether to not participate in the labor force. These workers are hired by profit-maximizing intermediate producers. Producers decide whether to maintain vacancies and, if so, in which of the two occupation/goods and abilities markets. Given equilibrium prices, outside options, and government policies, intermediate goods firms choose the optimal levels of vacancies. Free entry implies zero lifetime profits. We discuss firms' choices next, and worker's choices in Section 3.3.

3.2.1. Routine Intermediate Good Producers

In order to hire R workers who have idiosyncratic productivity ε_R, a firm must post vacancies v_{ε_R} at a flow cost κ_{ε_R} per vacancy. A constant returns to scale matching function, $M(v_{\varepsilon_R}, u_{\varepsilon_R})$, determines the number of new matches given vacancies posted and the number of unemployed job searchers (u_{ε_R}) in this good-ability-specific market.

As is standard, firms take the tightness ratio ($\theta_{\varepsilon_R} = \frac{v_{\varepsilon_R}}{u_{\varepsilon_R}}$) and the vacancy filling probability $q(\theta_{R,\varepsilon_R})$ as given. A matched worker and firm produce $y_R = f_R \times \varepsilon_R$ units of the R intermediate good, which is sold to the final good producer at the competitive price P_R per unit. The firm pays a bargained wage ω_{R,ε_R} to the worker. Thus the flow profit from a match is $P_R f_R \varepsilon_R - \omega_{R,\varepsilon_R}$.

Let x_{ε_R} denote the number of employed R workers with idiosyncratic productivity ε_R. To derive the optimality condition for vacancy creation, we assume–for expositional clarity–that there exists a representative good-ability-specific firm that chooses v_{ε_R} to solve:

$$J(x_{\varepsilon_R}, \Lambda) = \max_{v_{\varepsilon_R}} \left\{ (1 - T_\pi) \left[x_{\varepsilon_R} \left(f_R \times \varepsilon_R \times P_R - \omega_{\varepsilon_R} \right) - \kappa_{\varepsilon_R} v_{\varepsilon_R} \right] + \mathbb{E} \left[\Theta \times J \left(x'_{\varepsilon_R}, \Lambda' \right) \right] \right\}$$

subject to the law of motion:

$$x'_{\varepsilon_R} = (1 - \delta_M) x_{\varepsilon_R} + v_{\varepsilon_R} q(\theta_{\varepsilon_R})$$

where δ_M is the exogenous match separation probability. The first order condition implies the optimality condition for vacancy posting:[11]

[11] The use of a representative firm is for convenience only. An identical optimal condition can be derived when assuming a Bellman value for an open vacancy, a Bellman value for a filled job, and a zero profit condition

$$V_{R,\varepsilon_R} = -(1 - T_\pi) \kappa_{R,\varepsilon_R} + q(\theta_{R,\varepsilon_R}) E \left[\Theta \times J'_{R,\varepsilon_R} \right] = 0$$

$$\frac{\kappa_{\varepsilon_R}}{q\left(\theta_{\varepsilon_R}\right)} = E\left[\Theta\left[f_R \times \varepsilon_R \times P_R - \omega_{\varepsilon_R} + (1-\delta_M)\frac{\kappa_{\varepsilon_R}}{q\left(\theta'_{\varepsilon_R}\right)}\right]\right]$$

Overall, then, the equilibrium quantity of efficiency R input is given by

$$R = (1 - Pop_{NRC})\int_{\varepsilon_R^*}^{\infty}\int_{-\infty}^{\varepsilon_{NRM}(\varepsilon_R)} \frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R} + UN_{\varepsilon_R}} \times \varepsilon_R \times f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_{NRM} d\varepsilon_R \tag{4}$$

where Pop_{NRC} denotes the share in population of high skilled workers and $\frac{EMP_{\varepsilon_R v}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}$ denotes the employment rate per a given ability ε_R. As we show below in Section 4.2, the economy is characterized by an ability cutoff in the R and NRM occupations as well as a function that determines in which occupation a worker works conditional on participating in the labor force. In Equation (4) the term ε_R^* denotes the cutoff ability in R such that below it workers do not work in R and the function $\varepsilon_{NRM}(\varepsilon_R)$ denotes the cutoff in ability NRM for each ability ε_R such that below it workers work in R and not in NRM. The expression $\frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}$ is the employment rate per an ability ε_R which is multiplies by its efficiency, ε_R, and the density, $f(\varepsilon_R, \varepsilon_{NRM})$.[12]

3.2.2. Non-Routine Manual Intermediate Good Producers

The NRM market is identical in its structure to the R labor market and therefore obeys the same optimality principles. For brevity we do not repeat the exposition and simply adapt the vacancy posting optimality condition:

$$\frac{\kappa_{\varepsilon_{NRM}}}{q\left(\theta_{\varepsilon_{NRM}}\right)} = E\left[\Theta\left[f_{NRM} \times \varepsilon_{NRM} \times P_{NRM} - \omega_{\varepsilon_{NRM}} + (1-\delta_M)\frac{\kappa_{\varepsilon_{NRM}}}{q\left(\theta'_{\varepsilon_{NRM}}\right)}\right]\right]$$

Overall, then, the equilibrium quantity of efficiency Non-Routine-Manual input is given by

$$NRM = (1 - Pop_{NRC})\int_{\varepsilon_{NRM}^*}^{\infty}\int_{-\infty}^{\varepsilon_R(\varepsilon_{NRM})} \frac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}} + UN_{\varepsilon_{NRM}}} \times \varepsilon_{NRM} \times f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_R d\varepsilon_{NRM}$$

$$J_{R,\varepsilon_R} = (1 - T_\pi)\left[f_R \times \varepsilon_R \times P_R - \omega_{R,\varepsilon_R}\right] + (1-\delta_R) E\left[\Theta \times J'_{R,\varepsilon_R}\right]$$

[12] As with the case of capital taxation, because firm profits are taxed net of vacancy costs, there are no equilibrium effects of profit taxation on low-skilled job creation.

3.2.3. Non-Routine Cognitive Intermediate Good Producers

For simplicity and given that our interest is in the low-skilled market we assume that the NRC market has no matching frictions. Hence the problem of the intermediate producer who produces with NRC workers is a static one, which implies that the marginal product of an NRC worker equals her wage. That is, the equilibrium condition is given by

$$\omega_{NRC} = f_{NRC} \times P_{NRC}$$

3.3. Workers

We assume that workers derive utility from consumption and have a strictly increasing and concave utility function, $U(C)$. The government provides transfers to the unemployed and those who are not in the labor force.Workers discount future utility with discount factor $0 < \beta < 1$.

3.3.1. Non-Routine Cognitive Workers

High-skill/NRC workers supply labor inelastically, participate in a frictionless labor market, earn $\omega_{NRC} = P_{NRC} \times f_{NRC}$, and are taxed on labor income at rate $T_{e,NRC}$. The workers split their income between consumption and saving in the form of trading an asset B that represents claims to profits of the intermediate goods firms. Let B_{NRC} denote the beginning of period value of such claims (the sum of dividends and resale value) that are traded at price p. Then, NRC workers solve:

$$V_{NRC}\left(B_{NRC},\Lambda\right) = \max_{C_{NRC},B'} \left\{U\left(C_{NRC}\right) + \beta\left[\mathbb{E}V_{NRC}\left(B'_{NRC},\Lambda'\right)\right]\right\}$$
$$\text{s.t.: } C_{NRC} + p \times B'_{NRC} = \omega_{NRC} \times \left(1 - T_{e,NRC}\right) + p \times B_{NRC}$$

3.3.2. Routine and Non-Routine Manual Workers

Given their idiosyncratic abilities, unmatched low-skill agents simultaneous choose: whether or not to participate in the labor market, and (conditional on participating) in which occupational labor market to search.

Let $\varepsilon = (\varepsilon_R, \varepsilon_{NRM})$ denote the vector of (constant) idiosyncratic ability of a worker. Let $V_{e,R,\varepsilon}(\Lambda)$ denote the value of an employed R worker with ability vector ε, $V_{u,\varepsilon_R}(\Lambda)$ the value of an unemployed worker Similarly, denote the value of being an unemployed R worker by $V_{u,\varepsilon_{NRM}}(\Lambda)$, the value of being unemployed NRM worker by $V_{u,NRM,\varepsilon}(\Lambda)$, and the value of being out of the labor force by $V_{\varepsilon_O}(\Lambda)$.[13]

[13] As before, Λ denotes the collection of aggregate state variables that workers take parametrically.

A matched R worker can choose to stay employed (unless exogenously separated), become unemployed and search in either occupational market, or drop out of the labor force.[14] In the event of an exogenous separation (with probability δ_R) the worker chooses in which market to search for employment or to drop out of the labor force. The dynamic problem of an employed R worker is therefore:

$$
\begin{aligned}
V_{e,\varepsilon_R}(\Lambda) = \quad & \max_{C_{e,\varepsilon_R},B'} \begin{Bmatrix} U\left(C_{e,\varepsilon_R}\right) + \\ \beta\left(1-\delta_M\right) E\left[\max\left\{V_{e,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_{NRM}}\left(\Lambda'\right), V_{\varepsilon_O}\left(\Lambda'\right)\right\}\right] + \\ \beta\delta_M E\left[\max\left\{V_{u,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_{NRM}}\left(\Lambda'\right), V_{\varepsilon_O}\left(\Lambda'\right)\right\}\right] \end{Bmatrix} \\
& \text{s.t.} : C_{e,\varepsilon_R} = \omega_{\varepsilon_R}\left(1-T_{e,\varepsilon_R}\right)
\end{aligned}
$$

where C_{e,ε_R} denotes consumption, ω_{ε_R} denotes the wage, and T_{e,ε_R} is the income tax rate.

An unemployed R worker with ability ε meets a vacancy with probability $\mu\left(\theta_{\varepsilon_R}\right)$ and then faces a choice of labor status that is identical to that of an employed worker.[15] In case the worker remains unmatched, she can choose to stay unemployed in the same market, become unemployed in the other market, or drop out of the labor force. The dynamic problem of an unemployed worker is:

$$
\begin{aligned}
V_{u,\varepsilon_R}(,\Lambda) = \quad & \max_{C_{u,\varepsilon_R},B'} \begin{Bmatrix} U\left(C_{u,\varepsilon_R}\right) + \\ \beta\mu\left(\theta_{\varepsilon_R}\right) E\left[\max\left\{V_{e,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_{NRM}}\left(\Lambda'\right), V_{\varepsilon_O}\left(\Lambda'\right)\right\}\right] + \\ \beta\left(1-\mu\left(\theta_{\varepsilon_R}\right)\right) E\left[\max\left\{V_{u,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_{NRM}}\left(\Lambda'\right), V_{\varepsilon_O}\left(\Lambda'\right)\right\}\right] \end{Bmatrix} \\
& \text{s.t.} : C_{u,\varepsilon_R} = b \times \omega_{\varepsilon_R} \times \left(1-T_{u,\varepsilon_R}\right)
\end{aligned}
$$

where C_{u,ε_R} denotes consumption of the unemployed, b denotes the replacement rate, and T_{u,ε_R} is the income tax rate. The problem for NRM workers is identical in structure as just described, except with R-subscripts replaced by NRM-subscripts and vice versa.

A worker who is out of the labor force chooses whether to remain out of the labor force, or become unemployed in either R or NRM. We assume that the transfer to those outside of the labor force is constant and independent of ability. Hence, the dynamic problem is:

$$
\begin{aligned}
V_o(\Lambda) = \quad & \max_{C_o,B'} \left\{U\left(C_o\right) + \beta E\left[\max\left\{V_{u,\varepsilon_R}\left(\Lambda'\right), V_{u,\varepsilon_{NRM}}\left(\Lambda'\right), V_o\left(\Lambda'\right)\right\}\right]\right\} \\
& \text{s.t.} : C_o = b_o \times \left(1-T_b\right)
\end{aligned}
$$

where C_O denotes consumption, and b_o denotes transfer payments to workers who are out of the labor force.

[14] An employed worker can not switch immediately to employment in the other sector due to the search and matching frictions.

[15] Recall that the labor market is segmented by occupation (i.e. R or NRM) and individual ability.

3.4. Wage Bargaining

A match between an intermediate good firm and a worker generates a positive surplus that must be divided. As is common in the literature, we assume the Nash bargaining solution to this surplus division. The surplus for an employer from a match is the marginal value from employing an additional worker

$$\frac{\partial J\left(x_{\varepsilon_R},\Lambda\right)}{\partial x_{\varepsilon_R}} = (1-T_\pi)\left(f_R \times \varepsilon_R \times P_R - \omega_{\varepsilon_R}\right) + (1-\delta_M)\, E\left[\frac{\partial J\left(x'_{\varepsilon_R},\Lambda'\right)}{\partial x'_{\varepsilon_R}}\right]$$

The worker's outside option consists of his choice between R and NRM, as well as the option to be out of the labor force. Therefore, the surplus of an employed R worker with idiosyncratic ability ε_R is

$$\tilde{V}_{\varepsilon_R}(\Lambda) = V_{e,R,\varepsilon}(\Lambda) - \max\left\{V_{u,\varepsilon_R}(\Lambda), V_{u,\varepsilon_{NRM}}(\Lambda), V_{\varepsilon_O}(\Lambda)\right\}$$

Denoting the worker's bargaining weight by τ and the firms bargaining weight by $1-\tau$, the wage for a worker employed in R with ability ε_R is the solution to

$$\max_{\omega_{\varepsilon_R}} \left(\tilde{V}_{\varepsilon_R}(\Lambda)\right)^{\tau} \left(\frac{\partial J\left(x_{\varepsilon_R},\Lambda\right)}{\partial x_{\varepsilon_R}}\right)^{1-\tau}$$

and an analogue solution for wages of all other workers, conditional on their occupation of employment and ability. In section 4 we impose assumptions and functional forms that allow for a analytic solution for the resulting wage function.

3.5. Government Budget Constraint

Let U_{NRM} denote the measure of workers who are unemployed and whose occupation is NRM. Formally,

$$UN_{NRM} = (1-Pop_{NRC}) \int_{\varepsilon^*_{NRM}}^{\infty} \int_{-\infty}^{\varepsilon_R(\varepsilon_{NRM})} \frac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}} + UN_{\varepsilon_{NRM}}} \times f(\varepsilon_R, \varepsilon_{NR})\, d\varepsilon_R\, d\varepsilon_{NRM}$$

and hence the unemployment transfers to this group is given by

$$UI_{NRM} = (1-Pop_{NRC}) \int_{\varepsilon^*_{NRM}}^{\infty} \int_{-\infty}^{\varepsilon_R(\varepsilon_{NRM})} \frac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}} + UN_{\varepsilon_{NRM}}} \times b \times \omega(\varepsilon_{NRM}) \times f(\varepsilon_R, \varepsilon_{NRM})\, d\varepsilon_R\, d\varepsilon_{NRM}$$

and similarly

$$UI_R = (1-Pop_{NRC}) \int_{\varepsilon^*_R}^{\infty} \int_{-\infty}^{\varepsilon_{NRM}(\varepsilon_R)} \frac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R} + UN_{\varepsilon_R}} \times b \times \omega(\varepsilon_R) \times f(\varepsilon_R, \varepsilon_{NRM})\, d\varepsilon_{NRM}\, d\varepsilon_R$$

Finally, denoting by NLF the measure of the unskilled population that is outside of the labor force the government transfers to this group b_O per each member.

The government revenues are given by labor and profit taxation. Specifically, the labor tax collected from NRM workers is given by

$$Rev_{NRM} = (1 - Pop_{NRC}) \int_{\varepsilon^*_{NRM}}^{\infty} \int_{-\infty}^{\varepsilon_R(\varepsilon_{NRM})} \frac{EMP_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}} + UN_{\varepsilon_{NRM}}} \times T_{\varepsilon_{NRM}} \times \omega(\varepsilon_{NRM}) \times f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_R d\varepsilon_{NRM},$$

and similarly the tax revenues collected from R workers is given by

$$Rev_R = (1 - Pop_{NRC}) \int_{\varepsilon^*_R}^{\infty} \int_{-\infty}^{\varepsilon_{NRM}(\varepsilon_R)} \frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R} + UN_{\varepsilon_R}} \times T_{\varepsilon_R} \times \omega(\varepsilon_R) \times f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_{NRM} d\varepsilon_R.$$

Finally, the revenue from the tax on profits of the intermediate producers in the NRM and R sectors are given by

$$Rev_{\pi_{NRM}} = (1 - T_\pi)(1 - Pop_{NRC}) \int_{\varepsilon^*_{NRM}}^{\infty} \int_{-\infty}^{\varepsilon_R(\varepsilon_{NRM})} [x_{\varepsilon_{NRM}} (f_{\varepsilon_{NRM}} \times \varepsilon_{\varepsilon_{NRM}} \times P_{NRM} - \omega_{\varepsilon_{NRM}}) - \kappa_{\varepsilon_{NRM}} v_{\varepsilon_{NRM}}] \times f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_R d\varepsilon_{NRM}$$

$$Rev_{\pi_R} = (1 - T_\pi)(1 - Pop_{NRC}) \int_{\varepsilon^*_R}^{\infty} \int_{-\infty}^{\varepsilon_{NRM}(\varepsilon_R)} [x_{\varepsilon_R} (f_R \times \varepsilon_R \times P_R - \omega_{\varepsilon_R}) - \kappa_{\varepsilon_R} v_{\varepsilon_R}] \times f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_{NRM} d\varepsilon_R$$

and the tax on the final good producer is given by

$$Rev_\pi = T_\pi \left[Y - P_R Y_R - P_{NRM} Y_{NRM} - P_{NRC} Y_{NRC} - \phi_A \left(X'_A - (1 - \delta_A) X_A \right) - \phi_K \left(K' - (1 - \delta_K) K \right) \right].$$

We assume that the government cannot issue debt and hence at each point in time the following budget constraint holds

$$NLF \times b_o + UI_{NRM} + UI_R = Rev_R + Rev_{NRM} + Rev_\pi + Rev_{\pi_R} + Rev_{\pi_{NRM}}$$

3.6. Equilibrium

Given aggregate and sector specific productivities, the distribution of idiosyncratic productivities, exogenous prices of investment in physical capital and ICT capital, exogenous measures of NRC and non-NRC workers, and a government tax and transfer policies, a symmetric stationary equilibrium with Nash bargaining is a collection of:

1. Prices of intermediate goods: P_{NRC}, P_R, P_{NRM}
2. Wages: $\omega_{NRC}, \omega_{R,\varepsilon}, \omega_{NRM,\varepsilon}$

3. Tightness ratios: $\theta_{R,\varepsilon}, \theta_{NRM,\varepsilon}$

4. Price for claims on equity p

5. Consumption of workers

6. Demand for claims on equity

7. Employment of workers by sector and type

8. Quantities produced by intermediate goods producers

9. Quantities of intermediate goods demanded by the final good producer

10. Investment in physical capital and ICT capital

such that

1. Vacancy choices solve the intermediate producers dynamic problem

2. The demand for intermediate goods and the accumulation of K and X_A solve the final good producer's dynamic problem.

3. The demand for consumption and claims to equity solve the workers' dynamic problem

4. The value of entry is zero in all markets (free entry)

5. The market for each intermediate good clears, i.e. the sum of all units produced equals the demand by the final good producer

6. The market for final good clears, i.e. the sum of demands for consumption and investments equals the supply of the final good

7. Wages solve the Nash bargaining problem

8. The government's budget is balanced

9. Equity is held entirely by the homogenous NRC workers

10. Budget constraint of the government holds

4. Construction of Steady State Equilibrium

In this section we describe the construction of the steady state equilibrium of the model and highlight a set of sufficient assumptions that deliver, as in the data, stationary unemployment rates even in the presence of productivity changes. The three conditions are (i) a constant relative risk aversion (hereafter CRRA) utility function, (ii) vacancy costs that are proportional to productivity, (iii) no income source for workers that is not proportional to the wage (i.e., an unemployment benefit that is modeled as a replacement rate relative to the wage when employed).

4.1. Wages and tightness ratios

The equilibrium in the labor market for low-skill workers consists of a wage and a tightness ratio for each occupation-and-ability-specific submarket. Recall that the bargaining problem maximizes the Nash product

$$\max_{\omega_{\varepsilon_R}} \left(\tilde{V}_{\varepsilon_R}(\Lambda)\right)^{\tau} \left(\frac{\partial J\left(x_{\varepsilon_R},\Lambda\right)}{\partial x_{\varepsilon_R}}\right)^{1-\tau}$$

and that he surplus of an employed R worker with idiosyncratic ability ε_R is

$$\tilde{V}_{\varepsilon_R}(\Lambda) = V_{e,R,\varepsilon}(\Lambda) - \max\left\{V_{u,\varepsilon_R}(\Lambda), V_{u,\varepsilon_{NRM}}(\Lambda), V_{\varepsilon_O}(\Lambda)\right\}.$$

Since we focus on the steady state of the model, consumption of firm owners is constant over time, so $\Theta = \beta$. Then, s we show in Appendix 9.1, the resulting wage function for an arbitrary R worker with ability ε_R is

$$\omega_{\varepsilon_R} = f_R \times \varepsilon_R \times P_R - \frac{1-\tau}{\tau} \frac{U\left(C_{e,\varepsilon_R}\right) - U\left(C_{u,\varepsilon_R}\right)}{U'\left(C_{e,\varepsilon_R}\right)\left(1-T_{e,\varepsilon_R}\right) - U'\left(C_{u,\varepsilon_R}\right)\left(1-T_{u,\varepsilon_R}\right)b_{\varepsilon_R}} + \theta_{\varepsilon_R}\kappa_{\varepsilon_R}$$

The wage is an increasing function of the worker's marginal revenue product, $f_R\varepsilon_R P_R$, as well as labor market tightness, $\theta_{R\varepsilon}$, which reflects the outside option for the worker. Unlike the standard DMP model with risk neutrality, the wage is also affected by the utility and marginal utility differences between employed and unemployed workers.

To characterize some features of the equilibrium, and with an eye toward quantitative analysis, we make assumptions about the functional form for utility. We assume a CRRA utility function $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ and show in Appendix 9.1 that the wage function simplifies to

$$\omega_{\varepsilon_R} = \frac{1}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[f_R \times \varepsilon_R \times P_R + \theta_{R,\varepsilon_R}\kappa_{R,\varepsilon_R}\right]$$

Further characterization relies on the first order condition for vacancies in steady state

$$\frac{\kappa_{\varepsilon_R}}{q(\theta_{\varepsilon_R})} = \beta\left[f_R \times \varepsilon_R \times P_R - \omega_{R\varepsilon} + (1-\delta_M)\frac{\kappa_{\varepsilon_R}}{q(\theta_{R\varepsilon})}\right]$$

which implicitly solves for the tightness ratio in this submarket as a function of model parameters. We follow Pissarides (2000) and assume that the hiring cost κ_{ε_R} is proportional to the worker type's productivity (reflecting the notion that is is more costly to hire more productive workers). Thus,

$$\kappa_{\varepsilon_R} = f_R \times P_R \times \varepsilon_R \times \kappa_0$$

where $\kappa_0 > 0$ is an exogenous parameter. With this assumption, we show in Appendix 9.1 that the equilibrium tightness ratio implicitly solves

$$\kappa_0 = \frac{\beta\frac{\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}}{\frac{1-\beta(1-\delta_M)}{q(\theta_{R\varepsilon})} + \beta\frac{\theta_{R\varepsilon}}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}}$$

implying we can write the wage function as linear in the idiosyncratic ability. While this equation cannot be solved analytically, it is clear that the equilibrium tightness ratio is independent of the fundamental productivity parameters of the economy. As a result the model yields a constant tightness ratio for each occupation in steady state equilibrium, even as productivity (i.e. automation technology) varies. This is consistent with the empirical patterns of the unemployment rate discussed in Section 2. The result that tightness is independent of productivity parameters is also useful in establishing results regarding productivity cutoffs, which we discuss next.

4.2. Productivity cutoffs

In Appendix 9.2 we show that the steady state value for unemployment in occupations R and NRM, conditional on idiosyncratic productivities, can be expressed as

$$V_{u,\varepsilon_R} = \frac{(f_R \times P_R \times \varepsilon_R)^{1-\sigma}}{1-\beta} \times \urcorner_R(\varepsilon_R)$$

$$V_{u,\varepsilon_{NRM}} = \frac{(f_{NRM} \times P_{NRM} \times \varepsilon_{NRM})^{1-\sigma}}{1-\beta} \times \urcorner_{NRM}(\varepsilon_{NRM})$$

where $\urcorner_R(\varepsilon_R)$ and $\urcorner_{NRM}(\varepsilon_{NRM})$ are functions of exogenous parameters and the tightness ratio for each ability within each occupation defined as follows

$$\urcorner_R = \left[\begin{array}{l} \dfrac{\left(b_{\varepsilon_R} \frac{1+\theta_{\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}} (1-T_{u,\varepsilon_R})\right)^{1-\sigma}}{1-\sigma} + \\ \left(\dfrac{1+\theta_{\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\right)^{-\sigma} \left[(1-T_{e,\varepsilon_R})^{1-\sigma} - (1-T_{u,\varepsilon_R})^{1-\sigma} b_{\varepsilon_R}^{1-\sigma}\right] \theta_{\varepsilon_R} \dfrac{\tau}{1-\tau}\kappa_0 \end{array} \right]$$

$$\urcorner_{NRM} = \left[\begin{array}{l} \dfrac{\left(b_{\varepsilon_{NRM}} \frac{1+\theta_{\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}} (1-T_{u,\varepsilon_{NRM}})\right)^{1-\sigma}}{1-\sigma} + \\ \left(\dfrac{1+\theta_{\varepsilon_{NRM}}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\right)^{-\sigma} \left[(1-T_{e,\varepsilon_{NRM}})^{1-\sigma} - (1-T_{u,\varepsilon_{NRM}})^{1-\sigma} b_{\varepsilon_{NRM}}^{1-\sigma}\right] \theta_{\varepsilon_{NRM}} \dfrac{\tau}{1-\tau}\kappa_0 \end{array} \right]$$

We assume that (i) the tax rates are independent of ability within an occupation, (ii) the replacement ratios are constant across abilities, and (iii) the target tightness ratios are the same across abilities. Then, we establish the following results. First, recall that we assumed that the transfer to those outside of the labor force is constant and independent of ability resulting in the value of being outside of the labor force being constant. [16]Therefore, we can solve for the cutoff levels ε_R^* and ε_{NRM}^* such that a worker who draws an ability parameter ε below both cutoffs prefers to be out of the labor force. These cutoffs are given by

$$\varepsilon_R^* = \frac{b_o}{f_R \times P_R}\left(\frac{1}{\urcorner_R}\right)^{\frac{1}{1-\sigma}},$$

for searching for employment in the R market and

$$\varepsilon_{NRM}^* = \frac{1}{f_{NRM} \times P_{NRM}}\left(\frac{b_o}{\urcorner_{NRM}}\right)^{\frac{1}{1-\sigma}},$$

for searching for employment in the NRM market.

Those who draw ε such that only one ability is above the relevant cutoff will choose to participate in the labor force and in that occupation. Naturally, it is possible that a worker draws $\varepsilon = \{\varepsilon_R, \varepsilon_{NRM}\}$ such that the worker prefers searching for employment in either occupation over being out of the labor force. In this case, we can determine the sector that the worker chooses by comparing the value of unemployment in the two sectors. Specifically, for each draw of ε_R we can find a corresponding level $\hat{\varepsilon}_{NRM}$ such that if $\varepsilon_{NRM} < \hat{\varepsilon}_{NRM}$ (and $\varepsilon_R > \varepsilon_R^*$) then the worker chooses to search in occupation R, and if $\varepsilon_{NRM} \geq \hat{\varepsilon}_{NRM}$ (and $\varepsilon_{NRM} > \varepsilon_{NRM}^*$) then the worker searches in occupation NRM. This cutoff is the solution to

[16] In Section 6 we relax this assumption and explore the effect of changes in the transfers to those outside of the labor force.

$$\frac{(f_R \times P_R \times \varepsilon_R)^{1-\sigma}}{1-\beta} \times \daleth_R = \frac{(f_{NRM} \times P_{NRM} \times \varepsilon_{NRM})^{1-\sigma}}{1-\beta} \times \daleth_{NRM}$$

implying a linear function of the form

$$\hat{\varepsilon}_{NRM}(\varepsilon_R) = \left(\frac{\daleth_R}{\daleth_{NRM}}\right)^{\frac{1}{1-\sigma}} \frac{f_R P_R}{f_{NRM} P_{NRM}} \times \varepsilon_R$$

This result is technically important since in order to solve for the general equilibrium, we need to recover aggregate employment by occupation, as well as aggregate intermediate goods Y_R and Y_{NRM}. In order to construct these measures we are required to integrate over the probability distribution of ability. Given that (i) the integral bounds are linear and that (ii) the tightness ratios are stationary we can solve for the equilibrium allocations and welfare calculations in a simple closed form approach even though the model features (i) curvature in utility, (ii) curvature in production function, and (iii) frictions in the labor market.

5. Quantitative Results

In this section we calibrate the model economy and evaluate the impact of advancement in automation technology; we model this as a fall in the relative price of ICT capital, ϕ_A. As a guideline, we target pre-automation moments, feed in the observed change in the price of automation, and evaluate model performance by comparing 2017 predictions to observed US data.

5.1. Calibration

We begin this section by discussing the parametrization of the model. Table 3 lists the various parameters and their values.

Ability distribution As is common in the literature we assume that work ability is distributed jointly log normal. Hence, there are five parameters: two standard deviations, two means, and one correlation. Let σ_{ε_R} (μ_{ε_R}) be the standard deviation (mean) of the Routine ability, and σ_{NRM} (μ_{NRM}) be the standard deviation (mean) of the Non-Routine-Manual ability, and $\rho_{\varepsilon_R,\varepsilon_{NRM}}$ denotes the correlation between both abilities. We note that the model is "scale free": the means of the distribution are irrelevant and we normalize them to unity. The correlation between the two abilities cannot be identified in the data. As such, we begin by assuming that the $\rho_{\varepsilon_R,\varepsilon_{NRM}} = 0$. As we report in Appendix 10, the results of the model are almost identical with different correlations.

Table 3: Calibration

Parameter	**Value**	**Target**
Ability Distribution		
μ_{NRM}	1	Normalization
μ_R	1	
σ_{NRM}	0.9803	Occupations allocations variance of observed wages
σ_R	0.7436	
$\rho_{R,NRM}$	0	
Preferences		
β	0.9957	Monthly frequency; $r_{annual} = 0.05$
σ	1	log utility
Labor Market Frictions		
δ	0.02	Monthly exit rate 1989
τ	0.5	Petrongolo and Pissarides (2001)
Taxes and Transfers		
b_{NNRC}	0.5	Maximum allowed, US 1989
b_{NRC}	0.25	Median replacement ratio for fifth quartile Jaimovich et al. (2019*a*)
b_o	.0813	Marginal worker indifferent between NLF and unemployment
T_{NRM}	0.137	
T_R	0.137	Average group tax rate
T_{NRC}	0.267	
Depreciation Rates		
δ_K	0.06	Annual depreciation rates (see Eden and Gaggl (2018))
δ_A	0.19	
Prices of Capital		
ϕ_K	1	
ϕ_A	0.77	Eden and Gaggl (2018)
$\frac{\phi_A^{2017}}{\phi_A^{1989}}$	0.3244	Fall in ICT prices 1989-2017 (see Eden and Gaggl (2018))
Production Function: Shares		
η	.1099	
α	0.8154	Labor share, Routine Labor Share, ICT capital Income share, 1989
f_R	0.3022	
Production Function: Elasticities		
γ	0.31	Physical capital income share (see Eden and Gaggl (2018))
EOS_2	0	$\approx$constant NRM income share
ν	0.46	Split of R workers between NLF and NRM and $\Delta\frac{X_A}{EMP_R}$
EOS_1	-1.1	

We identify the standard deviations, σ_{ε_R} and σ_{NRM}, iteratively as follows. Given initial guesses for these two parameters, we find the ability cutoffs ε_R^* and ε_{NRM}^* such that the model implied allocation within the low-skill group identified from the ML approach matches the shares of R workers, NRM workers, and NLF in the data in 1989. We then check whether resulting variances of occupation-specific wages in the model match the observed variances in the CPS in 1989.

That is, following the discussion in Section 4 where we argued that the wage function is linear in the idiosyncratic ability and that the integral bounds are linear, then the log of the routine wage for an arbitrary ability ε_R can be written as

$$log\omega_{\varepsilon_R} = logD + log(\varepsilon_R)$$

where D denotes a fixed component that is identical to R workers of all abilities. This implies that the log of the R wage is distributed

$$\log \omega_{R,\varepsilon_R} \sim N\left(\mu_{\varepsilon_R} + \log D, \sigma_R\right),$$

and thus, the variance of the *observed* wages (for those above the cutoff) is given by

$$\text{Var}\left(\log \omega_{R,\varepsilon_R} | \log \varepsilon_R > \log \varepsilon_R^*\right) = \text{Var}\left(\log D + \log \varepsilon_R | \log \varepsilon_R > \log \varepsilon_R^*\right)$$

Given that D is a constant, this boils down to a variance in a truncated bivariate log normal

$$\text{Var}\left(\log \varepsilon_R | \log \varepsilon_R > \log \varepsilon_R^*\right),$$

with a similar expressions for the variance of observed NRM wages. We iterate on the guesses of the standard deviations until the resulting truncated wages in the model match those in the data (the standard deviation of the log observed wages for Routine workers in the data in 1989 is 0.237, while that for NRM equals 0.242).

Preferences There are two preference parameters: the discount rate, β, and the coefficient dictating relative risk aversion, σ. Since the model is calibrated to monthly job finding and exit rates, we set $\beta = 0.9957$ targeting an average annual risk free interest rate of 5%. We assume all individuals have logarithmic preferences in consumption, so that $\sigma = 1$.

Match separation rate We set the exogenous monthly separation rate, δ_M, equal to the 1989 rate of 0.02.

Labor matching function We assume a Cobb-Douglas matching function in each occupation-ability-specific market, with symmetric output elasticity with respect to vacancies and unemployed, equal to 0.5 (e.g., Petrongolo and Pissarides, 2001). Without loss of generality, we assume an identical matching efficiency across all markets equal to 1. We calibrate the tightness ratio to match an employment rate of 0.95 across all low-skill workers to match the evidence in Table 1.

Government transfers There are two types of government transfers in the model; unemployment insurance and transfers to those who are outside the labor force. With respect to the first one, recall that we model unemployment insurance benefits as replacement ratio. We set the replacement rate for all workers types to 0.5 which is the maximum allowed value in the U.S.[17]With respect to the second transfer, it it set internally to assure that when calibrated to match the 1989 labor market allocations (i.e. shares of R, NRM, and NLF) , the marginal worker is indifferent between participating in the labor force and being unemployed.[18]

Taxes Government transfers are funded by taxes on profit and labor income. We model the progressivity of the labor tax schedule by setting the tax on unemployment income and welfare transfers to be zero. With respect to the tax rate on NRM and R we set it at 13.7%, which is approximately the average tax rate across the second to fourth quintiles of income, while for NRC NRC occupations we set it at 26.7% which is the average federal tax rate for the fifth quintile of income.[19] We allow the profit tax rate, T_π, to adjust such that it balances the government budget constraint following the discussion in Section 3.5.

Depreciation rates We use the specific capital depreciation rates estimated by Eden and Gaggl (2018) and target an annual depreciation rate of $\delta_A = 19\%$ for ICT capital, and $\delta_K = 6\%$ on non-ICT, "physical" capital.

Relative prices of capital We use the same data to calibrate the two initial relative prices of capital to consumption to equal $\phi_K = 1$ and $\phi_A = 0.77$. As discussed above, our measure of automation is the change in the relative price of ICT capital between 1989 and 2017. Based on the estimate in Eden and Gaggl (2018) we feed in a fall in the ICT price such that $\frac{\phi_A^{2017}}{\phi_A^{1989}} = 0.3244$. This fall is our proxy for the decline in real price of automation, and hence, advancement in "automation technology".[20]

Production function: shares There are four parameters that control various income shares: η, α, τ, f_R. The moments we match are the shares of labor income, Routine labor income, and ICT capital income in

[17]We relax the assumption of a constant replacement rate, and discuss other types of transfers (including universal basic income) in Section 6.

[18]To provide context, the resulting value of consumption naturally equals the consumption value of the least able worker (either in Routine or Non-Routine Manual) and equals 0.37 of the average Routine wage.

[19]These tax rates are based on the estimates in the Congressional Budget Office distribution of household income in 2015.

[20]We note that the estimates in Eden and Gaggl (2018) end in 2013. We then extrapolate both the price series and capital series until 2017 based on the median growth rate in these two series in the post Great Recession period. As a robustness check we note that during period they overlap the relative chained price index of private fixed investment in information processing equipment and software behave in an almost identical way to the Eden and Gaggl (2018) series. See https://fred.stlouisfed.org/series/B679RG3Q086SBEA.

GDP, and an "internal consistency" moment for their 1989 values.[21]

The identification of these parameters is as follows: η and α determine the relatives shares that go to the different factors of production. τ splits the income generates in the labor market between operating profits and labor given the bargaining process. Finally, to identify f_R note that from the ratio of the aggregate income that goes to routine workers (S_R) and ICT capital (S_A) and from the firm's optimality condition we can recover the required value of f_R to be given by

$$f_R^{\nu} = \frac{\chi}{(1-\chi)} \frac{S_R}{S_A} \frac{X_A^{\nu}}{W_R \left(\frac{E}{u+U} \int_{\varepsilon_R^*}^{\infty} \int_{-\infty}^{\varepsilon_R + \log(m)} f(\varepsilon_R, \varepsilon_{NR}) d\varepsilon_{NR} d\varepsilon_R \right)^{\nu}}$$

Production function: elasticities I There are four parameters that control the production function elasticities; $\gamma, EOS_2, \nu, EOS_1$. With respect to γ, we assume that non-ICT capital has an elasticity of one with the composite of the other factors of production and thus we calibrate it directly from the Eden and Gaggl (2018) income share data to $\gamma = 0.31$. Similarly, with respect to EOS_2, as we discuss below the NRM share in national income has not changed during the period of interest. As such we assume it is also of Cobb Douglas form, and set $EOS_2 = 0$.

Production function: elasticities II The remaining two parameters are ν, which controls the elasticity of substitution between ICT capital and R labor services, and EOS_1, which controls the elasticity of substitution between NRC labor and the composite good of R labor services and ICT capital.

These two parameters cannot be identified from first moments and in order to calibrate them we proceed as follows. We feed in the empirically observed ICT price fall discussed above and iterate over ν and EOS_1 such that we match two moments: (i) our empirical result of the approximate two-third/one-third split between NLF and NRM (0.37) in accounting for the fall in R employment (more precisely, 0.63-0.37 split), and (ii) the change in the ratio of ICT capital per employed R worker between 1989 and 2017 (i.e the ratio of $\frac{X_A}{E_R}|_{2017}$ to $\frac{X_A}{E_R}|_{1989}$, which equals 7.14). We find that the model matches these two moments at values of $\nu = 0.46$ and $EOS1 = -1.1$. Reassuringly, we note that the model matches, without targeting it, the elasticity of ICT capital to its price; based on the ICT capital and relative price in Eden and Gaggl (2018), we calculate an elasticity of the stock of capital to its price over the 1989-2017 period of 0.4. The model under the above calibration generates an elasticity of 0.41.

[21] This internal consistency condition is such that given the ability cutoffs the following condition holds $m = \frac{\varepsilon_{NR}^*}{\varepsilon_R^*} = \frac{P_R F_R}{P_{NR} F_{NR}} \left(\frac{\urcorner_R}{\urcorner_{NR}} \right)^{\frac{1}{1-\sigma}}$. This is akin to an RBC model where the parameter of disutility of working is calibrated to match a given amount of working hours in the steady state.

5.2. Model Results

5.2.1. Allocations

To evaluate the empirical relevance of the model, and the role of ICT price change as a driving force in automation, we study its implications for several non-targeted data moments. Specifically, we analyze the implications regarding: (i) the fall in Routine employment propensity among the low-skilled, (ii) the change in the labor share of national income (and its occupational composition), and (iii) the behavior of relative average NRM and R wages. Column I in Table 4 reports these non-targeted moments in the model.

Likelihood of working in Routine With respect to the fall in the propensity of non-NRC (low-skill) individuals to work in Routine occupations the model generates a a fall of 7.85% log point fall. As discussed in Section 2, given the secular increase in female labor force participation, it is difficult to isolate the change in occupational employment and participation due to advances in automation for all low-skilled individuals. However, if we consider only non-NRC men between, 1 indicates a 15% log point fall in Routine employment, 1989-2017. The model generates over half of this. If we consider the non gender-specific fall between 1989 and 2017, then the model accounts for 35% of this propensity fall.[22]

National Income Shares Between 1989 and 2017, the share of GDP accruing as labor income fell by 4.3 percentage points (see, for e.g., Karabarbounis-Neiman, 2013). The model generates a fall of 2.39 percentage points, hence accounting for slightly more than half of the observed fall. With respect to the composition of labor income shares, Eden and Gaggl (2018) show that changes were not evenly distributed across occupations. Specifically, the routine occupational labor share of GDP has decreased dramatically between 1989 and 2017, by 9.51 percentage points, ie. more than twice the fall of aggregate labor's share of income. At the same time, the non-routine cognitive labor share has been rising by 4.17 percentage points, while the share of GDP accruing to non-routine manual employment has remained roughly constant as it increased by 0.67 percentage points.

As in the data, the fall in the share of GDP that goes to Routine workers is more than double the fall in aggregate labor; in the model this share falls by 6 percentage points. With respect to the income share that goes to NRC labor, the model yields an increase of roughly 3.5 percentage points, very close to the change observed in the data. Hence, to summarize, the model driven solely by a fall in the price of ICT capital as observed in the data, accounts for accounting for roughly half to two thirds of the overall fall in the labor

[22]Recall that, by construction, the model matches the way that the overall fall in R is split NLF and NRM.

income share as well as for the relative movements in the different occupation-specific components. [23]

Relative wages Finally, based on the CPS outgoing rotation groups, the relative average hourly wage of Non-Routine Manual to Routine workers rose by about 10 percent during our period of interest. [24]

What are the model predictions with respect to the behavior of relative wages? We first note that the the model generates a fall of 7.4% in the wage *per efficiency unit* of Routine workers, ω_R, and an increase of 4.2% in the wage per efficiency units of Non-Routine Manual workers, ω_{NRM}, resulting overall in a relative increase of 11%. These efficiency measures, of course, are not the empirically observed measures. As such, using the efficiency wages, equilibrium cutoffs, and the employment rates per type, we construct the average wage per Routine and Non-Routine manual workers as

$$E(\omega_R) = \frac{\omega_R \int_{\varepsilon_R^*}^{\infty} \int_{-\infty}^{\varepsilon_R + \log(m)} \frac{\mu(\varepsilon_R)}{\mu(\varepsilon_R)+\delta} \varepsilon_R f(\varepsilon_R, \varepsilon_{NR}) d\varepsilon_{NRM} d\varepsilon_R}{\int_{\varepsilon_R^*}^{\infty} \int_{-\infty}^{\varepsilon_R + \log(m)} f(\varepsilon_R, \varepsilon_{NR}) d\varepsilon_{NRM} d\varepsilon_R}$$

$$E(\omega_{NRM}) = \frac{\omega_{NR} \int_{\varepsilon_{NR}^*}^{\infty} \int_{-\infty}^{\varepsilon_{NR} - \log(m)} \frac{\mu(\varepsilon_{NRM})}{\mu(\varepsilon_{NRM})+\delta} \varepsilon_{NRM} f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_R d\varepsilon_{NRM}}{\int_{\varepsilon_{NRM}^*}^{\infty} \int_{-\infty}^{\varepsilon_{NRM} - \log(m)} f(\varepsilon_R, \varepsilon_{NRM}) d\varepsilon_R d\varepsilon_{NRM}}$$

In the model, the average R to NRM wage ratio increases by 3.6%, accounting for about a third of the observed change in the data.

5.2.2. Output and Welfare

What are the implications with respect to output? The model implies that the fall in ICT price and the resulting equilibrium allocations raise GDP by 11%. By way of comparison, output per capita has risen by about 40%, 1989-2017, in the data. Hence, the model implies, that about a quarter of the change of the observed output during this time period can be attributed to the drop in ICT prices.

What does this increase in output imply for the (consumption equivalence) change both in aggregate welfare as well as cross the different groups in the economy? In what follows we show that in spite the richness of the model our assumptions allow us to derive simple closed form solutions that characterize the welfare in the economy.

[23] Recall that we set NRM labor input to have an elasticity of substitution of 1 with the other factor of production implying that its share of national income does not change.

[24] A similar magnitude (a relative increase of about 12 percent) is observed in the average hourly wages constructed from the March annual earning supplement. We are grateful to Paul Gaggl for sharing this data with us.

Measures of welfare In order to construct the aggregate measure of welfare we first recall that the value of being unemployed in a R (NRM) occupation for an agent with a random ability ε_R (ε_{NRM}) is given by

$$V_{u,\varepsilon_R} = \varepsilon_R^{1-\sigma} \times \frac{(f_R \times P_R)^{1-\sigma}}{1-\beta} \times \urcorner_R$$

$$V_{u,\varepsilon_{NRM}} = \varepsilon_{NRM}^{1-\sigma} \times \frac{(f_{NRM} \times P_{NRM})^{1-\sigma}}{1-\beta} \times \urcorner_{NRM}$$

and the value of being employed is given by

$$V_{e,\varepsilon_R} = \frac{(1-\beta(1-\mu(\theta_{\varepsilon_R})))V_{u,\varepsilon_R} - \frac{\left(C_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})}$$

$$V_{e,\varepsilon_{MRM}} = \frac{(1-\beta(1-\mu(\theta_{\varepsilon_{NRM}})))V_{u,\varepsilon_{NRM}} - \frac{\left(C_{u,\varepsilon_{NRM}}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,NRM})}$$

Since we assume that non-NRC individuals are hand-to-mouth it follows that that the value of being employed is given by

$$V_{e,\varepsilon_R} = \left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_R}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})}\right)(f_R P_R \varepsilon_R)^{1-\sigma}$$

$$V_{e,\varepsilon_{MRM}} = \times\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_{NRM}}))\right)}{1-\beta}\urcorner_{NRM} - \frac{b_{\varepsilon_{NRM}}^{1-\sigma}\left(1-T_{u,\varepsilon_{NRM}}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,NRM})}\right)(f_{NRM} P_{NRM} \varepsilon_{NRM})^{1-\sigma}$$

Finally we note that the average welfare of an individual, while being in the labor force and in the R occupation, is then a weighted average of V_{u,ε_R} and V_{e,ε_R} where the weights are given by the unemployment and employment rate,

$$V_{\varepsilon_R} = \left[\begin{array}{l} \frac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\frac{\urcorner_R}{1-\beta} + \\ \frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_R}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})}\right)\end{array}\right](f_R P_R \varepsilon_R)^{1-\sigma}$$

and for the NRM,

$$V_{\varepsilon_{NRN}} = \left[\begin{array}{l} \frac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}}+UN_{\varepsilon_{NRM}}}\frac{\urcorner_{NRM}}{1-\beta} + \\ \frac{EMP_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}}+UN_{\varepsilon_{NRM}}}\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_{NRM}}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_{NRM}}^{1-\sigma}\left(1-T_{u,\varepsilon_{NRM}}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,NRM})}\right)\end{array}\right](f_{NRM} P_{NRM} \varepsilon_{NRM})^{1-\sigma}$$

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Then, the consumption equivalence utility for this agent is naturally given by

$$C_{\varepsilon_R} = \begin{bmatrix} \dfrac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R} + UN_{\varepsilon_R}} \dfrac{\urcorner_R}{1-\beta} + \\ \dfrac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R} + UN_{\varepsilon_R}} \left(\dfrac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_R}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})} \right) \end{bmatrix}^{\frac{1}{1-\sigma}} f_R P_R \varepsilon_R$$

$$C_{\varepsilon_{NRM}} = \begin{bmatrix} \dfrac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}} + UN_{\varepsilon_{NRM}}} \dfrac{\urcorner_{NRM}}{1-\beta} + \\ \dfrac{EMP_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}} + UN_{\varepsilon_{NRM}}} \left(\dfrac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_{NRM}}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_{NRM}}^{1-\sigma}\left(1-T_{u,\varepsilon_{NRM}}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,NRM})} \right) \end{bmatrix}^{\frac{1}{1-\sigma}} f_{NRM} P_{NRM} \varepsilon_{NRM}$$

Given our assumptions, the consumption equivalence value can then expressed as an aggregate component that is common within an occupation times a linear ability term. This significantly simplifies the welfare calculations since it allows us to simply use the realized abilities draws in order to calculate aggregate welfare terms as follows. Given the new equilibrium cutoff for $\varepsilon_R^{*,NEW}$ and $\varepsilon_{NRM}^{*,NEW}$ we simulate a billion individuals who draw their ability levels from the underlying joint log normal distribution. We then calculate the new measures or NLF, R, and NRM as follows

$$NLF^{NEW} = I\left(\varepsilon_R \leq \varepsilon_R^{*,NEW}\right) \times I\left(\varepsilon_{NRM} \leq \varepsilon_{NRM}^{*,NEW}\right)$$

$$NRM^{NEW} = I\left(log(m^{new}) + log(\varepsilon_R) \leq log(\varepsilon_2)\right) \times I\left(\varepsilon_{NRM}^{*,NEW} \leq \varepsilon_{NRM}\right)$$

$$R^{NEW} = I\left(log(m^{new}) + log(\varepsilon_R) > log(\varepsilon_2)\right) \times I\left(\varepsilon_R^{*,NEW} \leq \varepsilon_R\right)$$

where $I(.)$ is an indicator function and $m^{new} = \frac{\varepsilon_{NR}^{*,NEW}}{\varepsilon_R^{*,NEW}}$. We then identify those individuals who do not transition across labor states (i..e remain in their original occupation) as well as those who transition across labor states following the ICT relative price change. In this case there are three groups: (i) those used to be R and become NLF, (ii) those who used to be R and become NRM, and (iii) those who used to be NLF and are become NRM.[25] We proceed by deriving the change in the welfare of these different groups.

Previously Routine workers As discussed above, after the fall in the ICT price, previous Routine workers are composed of three groups: (i) those who remain employed in R, (ii) those who switch to NRM, and (iii)

[25] In the experiments reported in Section 6 there are additional labor state transitions.

those who drop out of the labor force. Those in Group (i) have a high enough ratio of R-vs-NRM ability and a high enough R ability vs the option of NLF participation.

$$\Delta_{R^{OLD}\to R^{NEW}} = \frac{\left[\frac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\frac{\urcorner_R}{1-\beta} + \frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_R}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})}\right)\right]^{\frac{1}{1-\sigma}} f_R P_R^{NEW} E\left(\varepsilon_R\right)^{R^{OLD}\to R^{NEW}}}{\left[\frac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\frac{\urcorner_R}{1-\beta} + \frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_R}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})}\right)\right]^{\frac{1}{1-\sigma}} f_R P_R^{OLD} E\left(\varepsilon_R\right)^{R^{OLD}\to R^{NEW}}} = \frac{P_R^{NEW}}{P_R^{OLD}}$$

where $E(\varepsilon_R)^{R^{OLD}\to R^{NEW}}$ denotes the average ability of those who remain in R. Hence, consumption of this group experiences the full fall in the per efficiency R wage, which translate (given the hand-to-mouth assumption) to a fall of 7.4% in its consumption equivalent welfare value. The second group, those whose NRM abilities are high enough such that they are better off in switching into NRM (and choose not to leave the labor force), experience an average consumption equivalence change in welfare that is given by

$$\Delta_{R^{OLD}\to NRM^{NEW}} = \frac{\left[\frac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}}+UN_{\varepsilon_{NRM}}}\frac{\urcorner_{NRM}}{1-\beta} + \frac{EMP_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}}+UN_{\varepsilon_{NRM}}}\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_{NRM}}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_{NRM}}^{1-\sigma}\left(1-T_{u,\varepsilon_{NRM}}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,NRM})}\right)\right]^{\frac{1}{1-\sigma}} f_{NRM} P_{NRM}^{NEW} E\left(\varepsilon_{NRm}\right)^{R^{OLD}\to NRM^{NEW}}}{\left[\frac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\frac{\urcorner_R}{1-\beta} + \frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\left(\frac{\frac{\left(1-\beta(1-\mu(\theta_{\varepsilon_R}))\right)}{1-\beta}\urcorner_R - \frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu(\theta_{\varepsilon,R})}\right)\right]^{\frac{1}{1-\sigma}} f_R P_R^{OLD} E\left(\varepsilon_R\right)^{R^{OLD}\to NRM^{NEW}}}$$

where we note that in the denominator we draw the ε_{NRM} abilities for these individuals that transitions to NRM. This group also experience a fall in welfare, albeit smaller than the first group; it amounts to a fall of 6.2% in consumption equivalence. Finally, the third group of previously R workers who leave the labor force experience an average welfare changes that is given by

$$\Delta_{R^{OLD}\to NRLF^{NEW}} = \frac{\frac{1}{1-\beta}\frac{1}{1-\sigma}\left(b_O\right)^{\frac{1}{1-\sigma}}}{\left[\frac{UN_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\frac{\urcorner_R}{1-\beta}+\frac{EMP_{\varepsilon_R}}{EMP_{\varepsilon_R}+UN_{\varepsilon_R}}\left(\frac{\frac{\left(1-\beta\left(1-\mu\left(\theta_{\varepsilon_R}\right)\right)\right)}{1-\beta}\urcorner_R-\frac{b_{\varepsilon_R}^{1-\sigma}\left(1-T_{u,\varepsilon_R}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu\left(\theta_{\varepsilon,R}\right)}\right)\right]^{\frac{1}{1-\sigma}} f_R P_R^{OLD} E\left(\varepsilon_R\right)^{R^{OLD}\to NLF^{NEW}}}$$

which we find to be equal to a fall of 4.4% in its consumption equivalent welfare.

Previously Non-Routine-Manual workers All previously Non-Routine-Manual workers remain working in that occupation. Hence, the change in the consumption equivalence for this group is given by

$$\Delta_{NRM^{OLD}\to NRM^{NEW}} = \frac{P_{NRM}^{NEW}}{P_{NRM}^{OLD}}$$

We find that the increase in ICT capital, which is a complement to NRM labor services in final production increases this ratio by 4.2%.

Previously NLF individuals After the fall in the ICT price, previous non-participants are composed of two groups: (i) those who remain outside of labor force, and (ii) those who enter NRM employment. The first group does not see a change in its welfare since government transfers do not change. The second group, composed of those individuals who have a sufficiently high NRM ability and respond to the increasing return to NRM labor services, sees an increase in its welfare which is given by

$$\Delta_{NLF^{OLD}\to NRM^{NEW}} = \left[\frac{UN_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}}+UN_{\varepsilon_{NRM}}}\frac{\urcorner_{NRM}}{1-\beta}+\frac{EMP_{\varepsilon_{NRM}}}{EMP_{\varepsilon_{NRM}}+UN_{\varepsilon_{NRM}}}\left(\frac{\frac{\left(1-\beta\left(1-\mu\left(\theta_{\varepsilon_{NRM}}\right)\right)\right)}{1-\beta}\urcorner_{NRM}-\frac{b_{\varepsilon_{NRM}}^{1-\sigma}\left(1-T_{u,\varepsilon_{NRM}}\right)^{1-\sigma}}{1-\sigma}}{\beta\mu\left(\theta_{\varepsilon,NRM}\right)}\right)\right]^{\frac{1}{1-\sigma}} \frac{f_{NRM}P_{NRM}^{NEW}E\left(\varepsilon_{NRM}\right)^{NLF^{OLD}\to NRM^{NEW}}}{\frac{1}{1-\beta}\frac{1}{1-\sigma}\left(b_O\right)^{\frac{1}{1-\sigma}}}$$

which we find to amount to 2.86%.

NRC workers Finally, not surprisingly, NRC workers benefit the most from the decline in the price of automation technology. These workers experience a consumption equivalent increase of 22.3%. There are two sources of this increase. First, the labor input of NRC is a complement with ICT capital. Second, this group holds all equity in the economy and thus benefits from the rise in firm profits.

Aggregate welfare Given the changes in the consumption equivalence across the different groups discussed above and their weights in population we can calculate the aggregate welfare. We find this to have increased by 3.17%.

6. Policy Experiments

In this section we consider a variety of government policies that carry labor market, aggregate, and distributional implications. For each policy, we analyze its equilibrium consequences both in terms of allocations and welfare.

We consider two sets of policies. First, we study the effects of policies that are aimed directly at counteracting the detrimental effects of ICT on the low-skilled. These include: (i) ICT taxation and (ii) a "retraining program", improving the work ability (in a distributional sense)of the low-skilled. Second, we consider a broader set of policies discussed in the contexts of inequality and redistribution, as well as in the contexts of "employment disappearance": (i) universal basic income (UBI hereafter), (ii) reforms to the unemployment insurance system, and (iii) changes in transfers to those outside of the labor force. Naturally, given the GE nature of the model, each of these policy experiments is required to be financed with increased government tax revenue. For simplicity, we assume that taxes on labor income remain unchanged and that only profit taxes are adjusted to maintain budget balance. This, naturally, has direct effects on the income of the high-skilled who hold firm equity.[26]

In order to anchor our analysis and be able to make allocational and welfare comparisons across steady state equilibria, we proceed as follows. For each of the policy experiments, we start the model at its 2017 obtained value (displayed in Column I), and search for the policy change such that the labor force participation of the low-skilled, non-NRC group returns to its 1989 "pre-automation" value. Columns II-VI in Table 4 reports the results of interest across the various policy experiments.

6.1. ICT Taxation

A tax on ICT capital purchases, τ_A, alters, in steady state, the ICT accumulation equation (3) to be:

$$(1+\tau_A)\phi_A = \frac{\beta}{(1-\beta(1-\delta_A))}MPA$$

[26] In Appendix 11 we report the results where the profit tax is unchanged and the taxes on labor are the ones that adjust to balance the government budget constraint.

Table 4: Policy Experiments

	I	**II**	**III**	**IV**	**V**	**VI**
Cutoffs						
$\Delta\varepsilon^*_{NRM}$; $\Delta\varepsilon^*_R$	-4.29 ; 9.64	4.29 ; 9.64	6.01 ; 0	-6.09 ; -8.07	-5.14 ; -7.08	-4.93 ; -7.02
Labor states and Shares						
ΔNLF (ΦNLF)	7.75 (2.57)	-7.75 (-2.57)	-7.75 (-2.57)	-7.75 (-2.57)	-7.75 (-2.57)	-7.75 (-2.57)
ΔR (ΦR)	-7.85 (-4.14)	7.85 (4.14)	1.02 (0.52)	3.73 (1.92)	3.80 (1.96)	3.80 (1.96)
ΔNRM (ΦNRM)	11.17 (1.57)	-11.17 (-1.57)	13.48 (2.13)	4.30 (0.65)	4.04 (0.61)	4.04 (0.61)
Emp Rate (R)	0.95	0.95	0.95	0.91	0.92	0.95
Emp Rate (NRM)	0.95	0.95	0.95	0.91	0.92	0.95
ΦLabor Share: Agg	-2.39	2.39	0	-2.01	-1.81	0.12
ΦLabor Share: R	-6.00	6.00	0	-0.75	-0.90	-0.09
ΦLabor Share: NRC	3.62	-3.62	0	-0.90	-0.55	0.21
ICT per R worker						
$\Delta\left(\frac{X_A}{\frac{EMP_R}{EMP_R+UN_R}\times R}\right)$	198	-198	8.74	3.72	1.22	-4.6
GDP and Profit Tax						
ΔGDP	11.18	-11.18	1.11	-3.53	-2.18	0.7
ΦT_π	-2.42	2.42	-3.69	1.29	-1.22	-5.93
Relative Wages						
$\Delta\omega_R$	-7.45	7.45	0	5.22	4.77	-1.39
$\Delta\omega_{NRM}$	4.22	-4.22	-6.01	5.49	3.91	-1.20
$\Delta\frac{E(\omega_R)}{E(\omega_{NRM})}$	-3.62	3.62	12.6	2.59	0.46	0.15
Welfare						
$\Delta_{R^{OLD}\to R^{NEW}}$	-7.2	7.2	1.00	2.91	1.48	-1.35
$\Delta_{R^{OLD}\to NRM^{NEW}}$	-1.7	1.7	1.00	NA	1.85	-1.27
$\Delta_{R^{OLD}\to NLF^{NEW}}$	-4.4	4.4	NA	NA	NA	NA
$\Delta_{NRM^{OLD}\to R^{NEW}}$	NA	NA	0.97	10.99	NA	NA
$\Delta_{NRM^{OLD}\to NRM^{NEW}}$	4.3	4.3	0.94	9.22	5.47	-1.18
$\Delta_{NRM^{OLD}\to NLF^{NEW}}$	NA	NA	0.96	NA	NA	NA
$\Delta_{NLF^{OLD}\to R^{NEW}}$	NA	NA	NA	9.16	3.80	-3.00
$\Delta_{NLF^{OLD}\to NRM^{NEW}}$	2.8	-2.8	1.12	8.08	3.93	-2.91
$\Delta_{NLF^{OLD}\to NLF^{NEW}}$	1	1	1	3.50	2	-6.00
$\Delta_{NRC^{OLD}\to NRC^{NEW}}$	22.22	-22.22	2.70	-4.84	-2.91	3.27
ΔAgg Welfare	3.17	-3.17	0.39	2.01	0.51	-1.32

Notes: Δ =log points change; Φ =percentage points change. Column I: ICT price change; Column II: ICT taxation; Column III: Change in ε_{NRM} distribution; Column IV: UBI ; Column V: UI ; Column VI :NLF benefits.

Hence, it is obvious that in order to induce the economy to return to 1989 allocations, all that is required is to institute a tax such that its product with the the 2017 ICT price equals the 1989 value of ϕ_A. Naturally, such a tax would exactly undo all of the effects of the fall in the price of ICT discussed in Section 5 (and the tax rate would have to continue to rise in the event that ϕ_A continues to drop). Hence, Column II in Table 4 shows that the tax rate that recovers the 1989 value of NLF would exactly undo all the effects of the ICT fall (vis-a-vis the 2017 values in Column I).

6.2. Retraining program

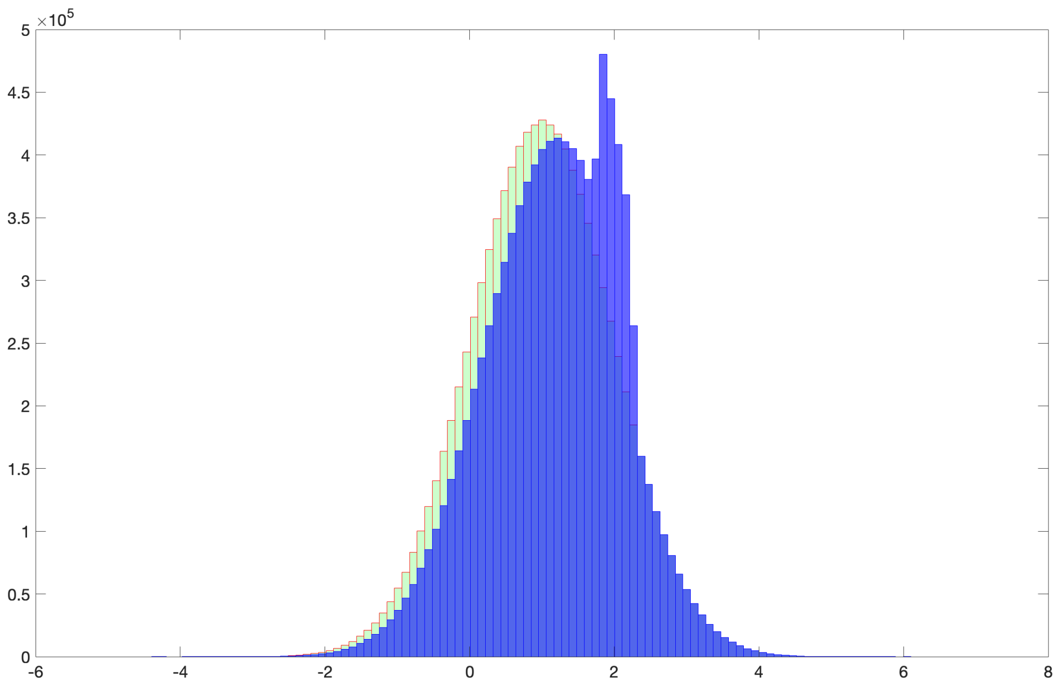
Our second policy experiment changes the abilities of low-skill workers in the face of automation. Specifically, we consider a change in the ε_{NRM} ability distribution, capturing the idea of training low-skill workers to do non-routine jobs. In this retraining policy, we target low-skill individuals who have selected into labor force non-participation (i.e. whose ability draws are below the cutoffs ε_R^* and ε_{NRM}^*) in the 2017, post-automation steady state (after the fall in the ICT price). Among those individuals, we shift the mean of the ε_{NRM} distribution such that the resulting NLF share of the low-skill group returns to the 1989 level. Since the experiment implies that the resulting ability distribution is no longer log normal we cannot rely on closed form solutions of the multivariate log-normal distribution. Rather we rely on numerical simulation of one billion individuals and calculate the resulting equilibrium. Figure 1 depicts the resulting ability distribution in blue while the original log-normal distribution is in green. Column III in Table 4 shows the effect vis-a-vis the values in Column I (the 2017 "post automation" steady state).

Labor market In order to increase the labor force participation and bring it back to its 1989 level, about 10% of those who are out of the labor force require an increase in their abilities that equals about a third of the standard deviation of NRM ability. This experiment results, not surprisingly, in a relative much larger increase into NRM than to R.

GDP This retraining policy results in an increase of GDP by slightly more than 1%. Since both labor force participation and productivity of NRM workers increase, government transfers–net of the retraining program cost–are *reduced* (note also that this policy does not affect the unemployment rate). This results in a fall in the equilibrium profit tax fall by 3.69 percentage points.

Capital stocks, pre-tax profits The retraining program increases supply of effective NRM workers. Given its complementarity with ICT, it increases the incentive to accumulate capital. At the same time it also

Figure 1: ε_{NRM} ability distribution



Notes: XXX

increases the marginal product in the R occupation. Overall, we find that the ratio of the ICT capital stock per routine worker increases by 8.74%

Welfare Economy wide, consumption equivalent welfare increases by 0.39%. The biggest beneficiaries of this policy are the high-skilled, NRC workers. They benefit both from the decline in the profit tax rate and from the increase in the the labor input of both R and NRM (since their labor is complementary to both the NRM and R workers).

With respect to the low-skill, non-NRC workers, those who receive the training see on average an increase in their consumption equivalence welfare of 12%. At the same time, those workers who were *already* working as NRM before the retraining program see a fall in their average consumption equivalence welfare of about 5%. This is due to a "displacement" or "crowding out" effect;[27] That is, the increase in the supply of NRM abilities leads to a fall in the efficiency price of their labor, resulting in a fall in the wage of the pre-existing NRM workforce. This leads to an exit from the labor force of workers with NRM ability close to the pre-retraining threshold. Finally, those individuals who, prior to the retraining, were working in R see essentially no change in their consumption equivalence welfare.

Cost-Benefit analysis Since the literature provides little guidance regarding the appropriate "production function" (and hence cost structure) of retraining programs, we abstract in the analysis above from this policy experiment's cost. Yet, it is instructive to provide a proxy in terms of cost-benefit analysis. Recall that the experiment induced an increase of flows from outside of the labor force for approximately 10% of those who were not participating, resulting in an increase of output of 1.11%. Hence, as long as the various cost channels of the program (i.e. labor, capital and potential increases in taxes) would amount per participant to less than 30% of GDP per capita, the retraining program, from an aggregate perspective has a positive return.[28]

6.3. UBI

Next, we consider the effects of a Universal Basic Income. We model the UBI as an identical lump sum transfer to each individual in the economy. Hence, the budget constraint of each individual in the economy includes a new term, $UBI > 0$; as an example, the budget constraint for a routine worker of type ε becomes

[27] See e.g. XXX

[28] Recall that we assume that profit taxes are adjusted to maintain budget balance. Hence, factoring in the cost of retraining would raise profit taxes and reduce the income/welfare gains to the high-skilled. Recall however, that because firm profits are taxed net of vacancy costs, there are no equilibrium effects on low-skilled job creation.

$C_{e,\varepsilon_R} = \omega_{\varepsilon_R}(1 - T_{e,\varepsilon_R}) + UBI$. Naturally, the introduction of a UBI modifies the government budget discussed in Section 3.5 constraint to

$$UBI + NLF \times b_o + UI_{NRM} + UI_R = Rev_R + Rev_{NRM} + Rev_{\pi} + Rev_{\pi_R} + Rev_{\pi_{NRM}},$$

i.e. the left hand side which denotes expenditures includes now a UBI payment for each individual in the (unit mass) population. Naturally, the other variables in the government budget constraint change following the experiment.

The introduction of an additive term to the individual's budget constraint implies that the linearity of the solution approach discussed in Section 4 is no longer applicable, so that: (i) each market (defined by ε_R and ε_{NRM} for routine and non-routine manual respectively) will have a different tightness ratio, and (ii) the occupation cutoffs are no longer linear functions of ability. In order to solve for the general equilibrium, we need to recover aggregate employment by occupation, as well as aggregate intermediate goods Y_R and Y_{NRM}. Hence, we must integrate over the probability distribution of ability, where the integral bounds are now non-linear functions.[29] With no closed form solutions to $\hat{\varepsilon}_{NRM}$ and for θ_{ε_R}, we resort to numerical integration. In the first step, we discretize the ability distribution, solving for θ_{ε_R} on each point of the grid. In the second step we approximate the required functions of ε_R (cutoffs, employment rates etc.) using splines. In the final step, we use the splines to obtain the required integral. We conduct this procedure twice, once over the support of ε_R and once over the support of ε_{NRM}. We take a similar approach to obtain welfare measures by occupation and by transition types.

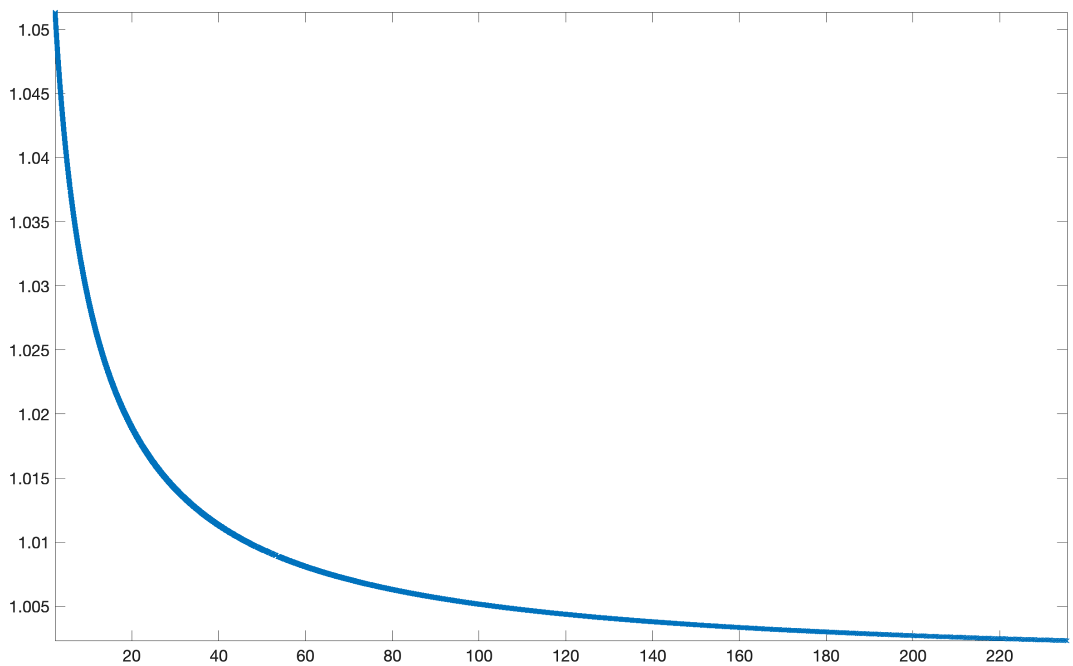
Column IV in Table 4 shows the effect vis-a-vis the values in column I (i.e. the state of the economy following the ICT price fall).

Labor market In order to increase the labor force participation rate among low-skill types and bring it back to its 1989 level, the required UBI equals a value that amounts to 1.34% of the average routine wage following the ICT price fall. The increase in labor force participation results in an split of about tow thirds into R occupations and one third in NRM occupations (1.92 vs 0.65 percentage points as Table 4 shows).

Why does the introduction of a *positive* UBI induce individuals to join the labor force? The key insight is that the UBI strengthens the bargaining position of workers, making the utility difference between working and unemployment much smaller. This in turn ends up increasing the Nash bargained wages, ceteris paribus (See Jaimovich et al. (2019*b*) for a general analysis of the general equilibrium effects of UBI in the presence of labor market frictions). The effects on the wage are depicted in Figure 2, which shows the ratio of the

[29] For example, the share of workers in R occupations corresponds to the mass of εs satisfying $\varepsilon_{NRM} < \hat{\varepsilon}_{NRM}$ (and $\varepsilon_R > \varepsilon_R^*$), and the share of employed in R will take into account the different tightness ratios by ε_R.

Figure 2: Wage figure

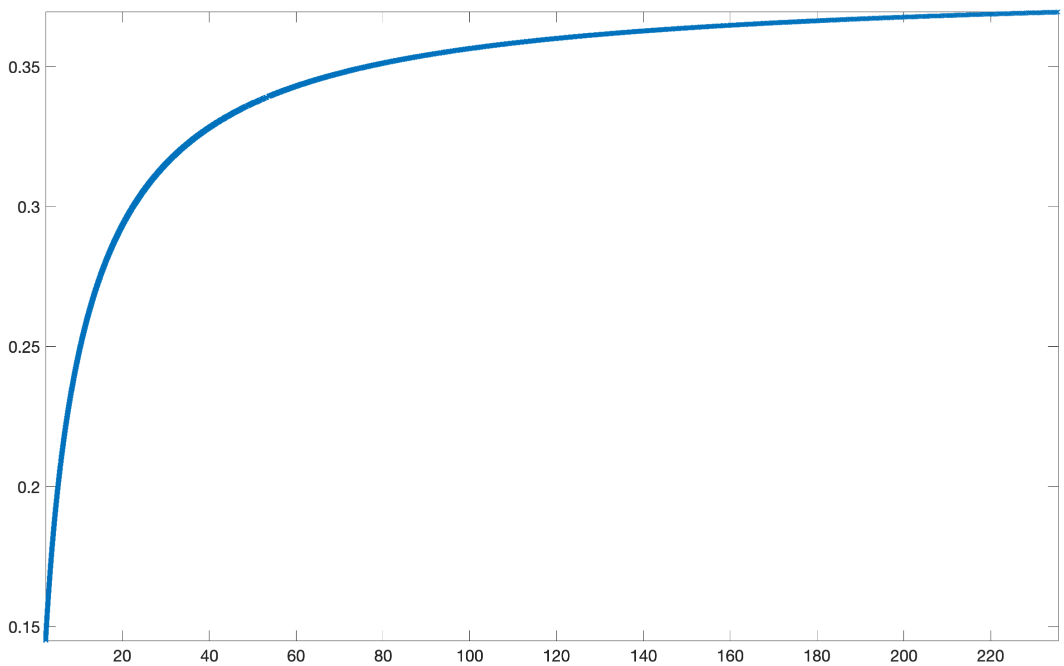


Notes: XXX

new equilibrium wage to the pre-UBI wage at each Routine ability level, ε_R. The post-UBI wage increases for each ability, though less at high ability levels as the UBI amounts to a smaller fraction of income. Moreover, since unemployment benefits are modeled as a replacement ratio, the increase in the equilibrium wage increases unemployment insurance payments, inducing further the incentives to participate in the labor force.

GDP While labor force participation increases, the introduction of the UBI *reduces* GDP by 3.5%. How can these effects coexist? To understand this, note that the increase in the bargaining position of workers (and the resulting wage increase) reduce firms' incentives to post vacancies at each ability level. This leads to a jump in unemployment. Figure 3 depicts the job finding rate for each Routine ability type; as a point of comparison, prior to the introduction of the UBI, the job finding rate was 0.38. This fall in the job finding rate is manifested in an increase in the unemployment rate which almost doubles from a baseline average of

Figure 3: Job Finding Rates



Notes: XXX

5% to almost 9%.

ICT per R worker The fall in the employment rate in R workers dominates the increase in the labor force participation. The fall in the factor of production that is substitutable to ICT capital, ceteris paribus increases the incentives to accumulate ICT capital. Indeed, we find that the introduction of the UBI increases the ratio of ICT capital per employed R by 3.7%.

Welfare Overall, despite the fall in GDP, the economy wide consumption equivalent welfare increases by 2.01%. This increase masks the distributional consequences of the UBI policy; for the high-skilled NRC workers, welfare decreases by 4.84%, while it increases between 3 and 10 percent for the low-skilled non-NRC workers.

The high-skilled, who hold firm equity in the economy, see a fall in after tax profits since greater profit taxation is required to fund the government transfers. Specifically, the increase in the unemployment rate and the higher unemployment benefits (due to the increases in wages) dominates the reduction in the the fraction of the population that is not in the labor force, resulting in an increase of 1.29 percentage points in the profit tax. Moreover, since the NRC are complementary with NRM and Routine labor, NRC productivity and wages fall, given the fall in employment in R and NRM occupations. These two effects dominate the direct income increase from the UBI.

With respect to the low-skilled, UBI accounts for a much greater share of their consumption compared to the NRC. The increase in bargaining position and wages, unemployment benefits, and the direct UBI transfer dominate the increase in the unemployment rate. As Column IV in Table 4 shows, all groups who are unskilled significantly benefit from the introduction of the UBI policy even though overall total output significantly falls.

6.4. Unemployment Insurance Benefits

In our next set of experiments we consider the effects of a change in the unemployment insurance (UI). We model this change as an identical lump sum transfer to each *unemployed* individual in the economy in addition to the existing unemployment benefits given via the replacement ratio (in contrast to the UBI that was given to all individuals in the economy). Hence, as an example the budget constraint of an unemployed individual for a routine worker of type ε becomes $C_{e,\varepsilon_R} = b_{\varepsilon_R}\omega_{\varepsilon_R}(1-T_{u,\varepsilon_R})+UI$.

The addition of an additive term to the budget constraint of the unemployed imply that the linearity of the solution approach discussed in Section 4 is no applicable. We follow the same solution approach in Section 6.3. Column V in Table 4 shows the effect vis-a-vis the values in Column I (i.e. the state of the economy following the ICT price fall).

Labor market In order to increase the labor force participation rate among low-skill types and bring it back to its 1989 level, the required UI transfer amounts to 0.38% of the average R wage following the ICT price fall. Similar to the UBI case, the increase in labor force participation is reflected in two thirds entry into R occupations and a third into NRM occupation.

As in the case of the UBI analysis above, the increase in the UI strengthens the bargaining position of workers, making the utility difference between working and unemployment much smaller, resulting in higher wages.

GDP As in the case of the UBI, the introduction of the UI transfer reduces GDP, this time by 2.18%. Overall, job findings rates fall, manifested in an increase in the unemployment from a baseline average of 5% to 8%.

ICT per R worker The fall in the employment rate in R workers dominates the increase in the labor force participation. The fall in the factor of production that is substitutable to ICT capital, ceteris paribus increases the incentives to accumulate ICT capital. Indeed, we find that the introduction of the UI increases the ratio of ICT capital per employed R by 1.2%.

Welfare Despite the fall in GDP, overall consumption equivalent welfare increases by 0.51%. Welfare increases by measures that range between 1.5 and 5.5 percent for the low-skilled non-NRC workers.

The high-skilled see a reduction in the profit taxation that is required to fund government transfers. This is because increased labor force participation reduces the transfers to those who are outside the labor force; this reduces overall government transfers payments (the increase to the unemployed). Nonetheless, welfare decreases by 2.91% for the high-skilled, NRC types. This is because the increase in the unemployment rate of low-skilled workers means a fall in the wage of the NRC, due to their complementary in production.

With respect to the low-skilled, the increase the UI transfers, and its general equilibrium effects on wages, dominate the increase in the unemployment rate. As Column V in Table 4 shows, all groups who are unskilled significantly benefit from the introduction of the UI policy (albeit less than with the introduction of the UBI program).

6.5. Transfers to outside of the labor Force

Our last policy experiment is one where we search for the required reduction in benefits for those outside of labor force, to induce them to participate at 1989 levels.

Labor market In order to increase the labor force participation rate among low-skill types and bring it back to its 1989 level, we find that the required magnitude is a reduction of 6%.

GDP Naturally, this increase in the labor force results in a boost to aggregate output, resulting in an increase of 0.7%.

ICT per R worker The increase in the measure of R workers results in lower incentives to accumulate ICT capital, resulting in a fall of 4.6% in the ratio of ICT capital per R worker.

Welfare Overall, despite the increase in GDP, the economy wide consumption equivalent welfare falls by 1.32%. The high-skilled, see a strong reduction in the profit taxation that is required to fund the government transfers of almost 6 percentage points. Moreover, the increase in the supply of R and NRM workers increase the marginal productivity of the high-skilled NRC workers. These two effects results in an increase in consumption equivalence welfare of 3.27%.

For the low-skilled who were initially outside of the labor force the fall in transfers them naturally reduces their welfare. Similarly, to those low skilled workers who were participating, the transition of other unskilled into the labor force reduces their welfare via the general equilibrium effect of having more R and NRM workers leading to lower wages. As Column VI in Table 4 shows, all groups who are unskilled see their welfare measures fall significantly (especially those who remain non-participants after the policy experiment), even though total output increases.

7. Conclusions

We consider the dramatic change in the occupational composition of employment–specifically, the disappearance of middle-wage "routine employment"–observed over the past 35 years. We study the role of advancement in automation technology in generating these occupational changes, and its implications for labor market participation, macroeconomic outcomes, and the distribution of income and inequality. We find that for individuals who were most likely to work in routine occupations, the decline in such job opportunities were offset by increased likelihood of both labor force non-participation and employment in low-wage NRM occupations, with the former outcome exceeding the latter on the order of 2-to-1.

We develop a macroeconomic model with investment in automation capital, labor force and occupational choice, and heterogeneity in income and asset market access. When subjected to the empirically observed change in the relative price of ICT capital, the model accounts for about half of the key relevant macroeconomic phenomena including the decline in routine employment, changes in occupation-specific relative wages, and the decline in labor's share of national income and its occupational composition.

We use this model to study the aggregate and distributional impact of various government policies. In all cases, our policy experiments are specified to return the fraction of low-skilled workers to labor force participation in the "post-automation" equilibrium to the participation rate observed "prior to automation," while maintaining government budget balance. We find, for instance, that a retraining program that enhances the NRM ability of low-skill workers has positive aggregate income effects in the aggregate, and are beneficial for a large fraction of the population. From a cost/benefit perspective, such a policy is worthwhile as long as the per capita cost of retraining is less than a third of output per capita. By contrast, the introduction of a UBI or UI has a negative impact on aggregate income and raises unemployment, while overall increasing the welfare for the low-skilled part of the population. This is due to the improved outside option to employment faced by workers, which raises equilibrium Nash bargained wages, making job search more attractive and job creation less attractive. Finally when considering a program that reduces the transfers to those outside of the labor force we find this program to raise aggregate output while reducing the welfare of the unskilled and increasing it for the high-skilled.

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8. Appendix: Data

8.1. Data

8.1.1. CPS Data construction

We adopt the occupational classification system used in Jaimovich and Siu (2012) that affords ease of data access and replication. The classification is based on the categorization of occupations in the 2000 Standard Occupational Classification system. Non-routine cognitive workers are those employed in “management, business, and financial operations occupations” and “professional and related occupations”. Routine cognitive workers are those in “sales and related occupations” and “office and administrative support occupations”. Routine manual occupations are “production occupations”, “transportation and material moving occupations”, “construction and extraction occupations”, and “installation, maintenance, and repair occupations”. Non-routine manual occupations are “service occupations”. Detailed information on 3-digit occupational codes are available from the authors upon request.

8.1.2. Classification errors

Our ML approach classifies each person (at each point in time) into one of the four “likely” occupational groups (NRC, RC, NRM, and RM). However we present our main results aggregating to two workers types – NRC and non-NRC, hence Tables 5 and 6 show the confusion matrices for those two categories, separately for men and women respectively. In each matrix we add the precision (share of correctly classified objects within a predicted category) and recall (share of observed that were picked up by the prediction within a category) values.

Table 5: Confusion Matrix - Men

		Classified		
		NRC	non-NRC	Precision
True	NRC	506,002	294,252	**63.23%**
	non-NRC	242,256	1,213,131	**83.35%**
	Recall	**67.62%**	**80.48%**	

Table 6: Confusion Matrix - Women

		Classified		
		NRC	non-NRC	Precision
True	NRC	342,362	150,507	**69.46%**
	non-NRC	241,376	1,167,622	**82.87%**
	Recall	**58.65%**	**88.58%**	

8.1.3. Recovering true series from series with errors

The classification errors discussed in 8.1.2 imply that we do not have "clean" series for the dynamics of NRC and non-NRC type persons. However, we show now that while we cannot recover correct the classification a the individual level, it is possible to correct the aggregate series of interest. Suppose that we are interested in recovering the share or persons of NRC and non-NRC types in specific labor force status, and call these x_{NRC}, and x_{NNRC}. Define our observed values from the classifier as $\hat{x}_{NRC}$, and $\hat{x}_{NNRC}$, and define the classification outcomes in terms of the following shares (with the convention $S_{True|Classified}$) as in Table 7:

Table 7: Classification Definitions

		Classified	
		NRC	non-NRC
True	NRC	$S_{NRC\|NRC}$	$S_{NRC\|NNRC}$
	non-NRC	$S_{NNRC\|NRC}$	$S_{NNRC\|NNRC}$

Table 8: Labor market status and occupation composition changes 1989-2017 by type: Women

	non-NRC		NRC	
	(1)	(2)	(3)	(4)
	1989	2017	1989	2017
Population Weight	0.76	0.57	0.24	0.43
Fraction in R	0.41	0.30	0.12	0.12
Fraction in NRM	0.15	0.20	~0	0.01
Fraction in NRC	0.02	0.05	0.74	0.72
Fraction in NLF	0.39	0.41	0.13	0.14
Fraction in Unemployment	0.03	0.04	0.01	0.02
Unemployment rate	0.06	0.07	0.01	0.01

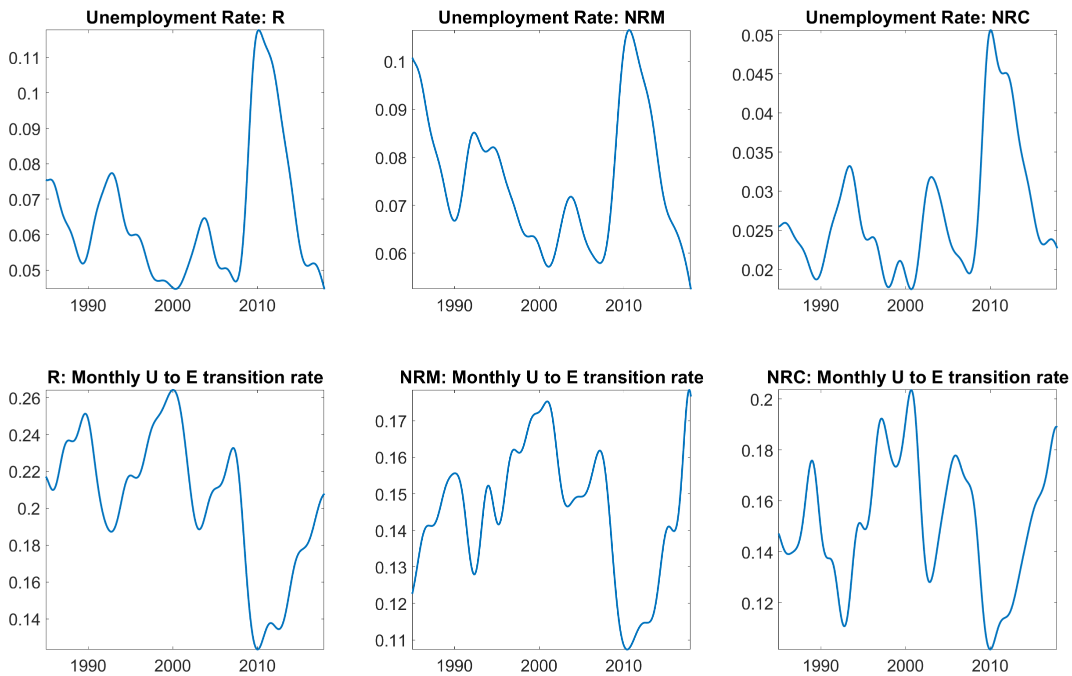
We can then write the observed values as a function of the true values and the share as follows

$$\begin{aligned}\hat{x}_{NRC} &= S_{NRC|NRC} x_{NRC} + S_{NNRC|NRC} x_{NNRC} \\ \hat{x}_{NNRC} &= S_{NRC|NNRC} x_{NRC} + S_{NNRC|NNRC} x_{NNRC}\end{aligned}$$

Thus if we know the shares in 7, we are left with a simple two-equation two-unknown linear system that will allow us to recover x_{NRC} and x_{NNRC}. The first way to recover the shares in 7 is to use the classification errors from the training, reported in section 8.1.2. The second approach is to use the restrictions implied by nature by some of the series. For example, the series or true values of employment share in R occupations for the NRC type *during the training period*, should be roughly zero. While the second approach is appealing, it can only be applied to the occupation series, and not to the NLF series, for which we apply the first approach. It is important to note that both approaches require the assumption that the classification errors are not correlated with the labor market status and occupation choice in the post-training period.

8.1.4. Unemployment and Exit Rates by Occupation

Figure 4: Unemployment and Exit Rates by Occupation



Notes:

9. Appendix: Model Derivations

9.1. Wage functions

Taking the first order condition with respect to wages we have

$$\tau\left(\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}\right)\left[U'\left(C_{e,R,\varepsilon}\right)\left(1-T_{e,R,\varepsilon}\right)-U'\left(C_{u,R,\varepsilon}\right)\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}\right]=\left(1-\tau\right)\left(\tilde{V}_{R,\varepsilon}\left(\Lambda\right)\right)\left(1-T_{\pi}\right)$$

or

$$\begin{aligned}\tilde{V}_{R,\varepsilon}\left(\Lambda\right)&=\left[U'\left(C_{e,R,\varepsilon}\right)\left(1-T_{e,R,\varepsilon}\right)-U'\left(C_{u,R,\varepsilon}\right)\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}\right]\frac{\tau}{1-\tau}\frac{1}{1-T_{\pi}}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}\\&=\xi\frac{\tau}{1-\tau}\frac{1}{1-T_{\pi}}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}\end{aligned}$$

Where $\xi\equiv\left[U'\left(C_{e,R,\varepsilon}\right)\left(1-T_{e,R,\varepsilon}\right)-U'\left(C_{u,R,\varepsilon}\right)\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}\right]$. Substituting for the marginal value of workers, and using the first order condition one period ahead, we can right the left hand side as

$$\begin{aligned}&\tilde{V}_{R,\varepsilon}\left(\Lambda\right)=U\left(\omega_{R,\varepsilon}\left(1-T_{e,R,\varepsilon}\right)\right)-U\left(b_{R,\varepsilon}\times\omega_{R,\varepsilon}\times\left(1-T_{u,R,\varepsilon}\right)\right)+\beta\left(1-\delta_R-\mu\left(\theta_{R,\varepsilon_R}\right)\right)\tilde{V}_{R,\varepsilon}\left(\Lambda'\right)=\\&=U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)+\beta\left(1-\delta_R-\mu\left(\theta_{R,\varepsilon_R}\right)\right)\xi\frac{\tau}{1-\tau}\frac{1}{1-T_{\pi}}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}\end{aligned}$$

Substitute for the marginal value of the firm we can write the right hand side as follows:

$$\begin{aligned}&\xi\frac{\tau}{1-\tau}\frac{1}{1-T_{\pi}}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}=\\&\xi\frac{\tau}{1-\tau}\frac{1}{1-T_{\pi}}\left[\left(1-T_{\pi}\right)\left(f_R\times\varepsilon_R\times P_R-\omega_{R,\varepsilon_R}\right)+\left(1-\delta_R\right)\beta\frac{\partial J\left(x'_{R,\varepsilon_R},\Lambda'\right)}{\partial x'_{R,\varepsilon_R}}\right]\end{aligned}$$

Therefore we have

$$U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)+\beta\left(1-\delta_R-\mu\left(\theta_{R,\varepsilon_R}\right)\right)\xi\frac{\tau}{1-\tau}\frac{1}{1-T_\pi}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}=$$
$$\xi\frac{\tau}{1-\tau}\frac{1}{1-T_\pi}\left[\left(1-T_\pi\right)\left(f_R\times\varepsilon_R\times P_R-\omega_{R,\varepsilon_R}\right)+\left(1-\delta_R\right)\beta\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}\right]$$
$$\Rightarrow$$
$$U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)-\beta\mu\left(\theta_{R,\varepsilon_R}\right)\xi\frac{\tau}{1-\tau}\frac{1}{1-T_\pi}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}=$$
$$\xi\frac{\tau}{1-\tau}\left(f_R\times\varepsilon_R\times P_R-\omega_{R,\varepsilon_R}\right)$$
$$\Rightarrow$$
$$\frac{1-\tau}{\tau}\frac{1}{\xi}\left(U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)\right)-\beta\mu\left(\theta_{R,\varepsilon_R}\right)\frac{1}{1-T_\pi}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}=$$
$$f_R\times\varepsilon_R\times P_R-\omega_{R,\varepsilon_R}$$
$$\Rightarrow$$
$$\omega_{R,\varepsilon_R}=f_R\times\varepsilon_R\times P_R-\frac{1-\tau}{\tau}\frac{1}{\xi}\left(U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)\right)+\beta\theta_{R,\varepsilon_R}q\left(\theta_{R,\varepsilon_R}\right)\frac{1}{1-T_\pi}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}$$

where we substitute the relationship $\mu\left(\theta_{R,\varepsilon_R}\right)=\theta_{R,\varepsilon_R}q\left(\theta_{R,\varepsilon_R}\right)$. Finally, we can use the steady state version of the first order condition for vacancies $\left(1-T_\pi\right)\kappa_{R,\varepsilon_R}=E\left[\beta q\left(\theta_{R,\varepsilon_R}\right)\frac{\partial J\left(x'_{R,\varepsilon_R},\Lambda'\right)}{\partial x'_{R,\varepsilon_R}}\right]$. This yields the general wage function

$$\omega_{R,\varepsilon_R}=f_R\times\varepsilon_R\times P_R-\frac{1-\tau}{\tau}\frac{1}{\xi}\left(U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)\right)+\theta_{R,\varepsilon_R}\kappa_{R,\varepsilon_R}=$$
$$f_R\times\varepsilon_R\times P_R-\frac{1-\tau}{\tau}\frac{U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)}{U'\left(C_{e,R,\varepsilon}\right)\left(1-T_{e,R,\varepsilon}\right)-U'\left(C_{u,R,\varepsilon}\right)\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}}+\theta_{R,\varepsilon_R}\kappa_{R,\varepsilon_R}$$

When we assume a CRRA utility function $U\left(C\right)=\frac{C^{1-\sigma}}{1-\sigma}$ and that there are no lump sum transfers to workers who are in the labor force then we can simplify further:

$$
\begin{aligned}
&\frac{U\left(C_{e,R,\varepsilon}\right)-U\left(C_{u,R,\varepsilon}\right)}{U'\left(C_{e,R,\varepsilon}\right)\left(1-T_{e,R,\varepsilon}\right)-U'\left(C_{u,R,\varepsilon}\right)\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}}=\\
&\frac{\frac{\left(C_{e,R,\varepsilon}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(C_{u,R,\varepsilon}\right)^{1-\sigma}}{1-\sigma}}{\left(C_{e,R,\varepsilon}\right)^{-\sigma}\left(1-T_{e,R,\varepsilon}\right)-\left(C_{e,R,\varepsilon}\right)^{-\sigma}\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}}=\\
&\frac{1}{1-\sigma}\frac{\left(\omega_{R,\varepsilon_R}\left(1-T_{e,R,\varepsilon}\right)\right)^{1-\sigma}-\left(b_{R,\varepsilon}\omega_{R,\varepsilon_R}\left(1-T_{u,R,\varepsilon}\right)\right)^{1-\sigma}}{\left(\omega_{R,\varepsilon_R}\left(1-T_{e,R,\varepsilon}\right)\right)^{-\sigma}\left(1-T_{e,R,\varepsilon}\right)-\left(b_{R,\varepsilon}\omega_{R,\varepsilon_R}\left(1-T_{u,R,\varepsilon}\right)\right)^{-\sigma}\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}}=\\
&\frac{1}{1-\sigma}\frac{\left(\omega_{R,\varepsilon_R}\right)^{1-\sigma}\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(\omega_{R,\varepsilon_R}\right)^{1-\sigma}\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}}{\left(\omega_{R,\varepsilon_R}\right)^{-\sigma}\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(\omega_{R,\varepsilon_R}\right)^{-\sigma}\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}}=\\
&\frac{1}{1-\sigma}\frac{\left(\omega_{R,\varepsilon_R}\right)^{1-\sigma}\left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}\right]}{\left(\omega_{R,\varepsilon_R}\right)^{-\sigma}\left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}\right]}=\\
&\frac{1}{1-\sigma}\omega_{R,\varepsilon_R}
\end{aligned}
$$

and as a result the wage function simplifies to

$$
\begin{aligned}
&\omega_{R,\varepsilon_R}=f_R\times\varepsilon_R\times P_R+\theta_{R,\varepsilon_R}\kappa_{R,\varepsilon_R}-\frac{1-\tau}{\tau}\frac{1}{1-\sigma}\omega_{R,\varepsilon_R}\\
&\Rightarrow\\
&\omega_{R,\varepsilon_R}=\frac{1}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[f_R\times\varepsilon_R\times P_R+\theta_{R,\varepsilon_R}\kappa_{R,\varepsilon_R}\right]
\end{aligned}
$$

Armed with this wage function we move to the optimality condition for vacancies

$$
\frac{\kappa_{R,\varepsilon_R}}{q\left(\theta_{R,\varepsilon_R}\right)}=\beta\left[f_R\times\varepsilon_R\times P_R-\omega_{R,\varepsilon_R}+\left(1-\delta_R\right)\frac{\kappa_{R,\varepsilon_R}}{q\left(\theta_{R,\varepsilon_R}\right)}\right]
$$

Substituting the wage function we have

$$
\frac{\kappa_{R,\varepsilon_R}}{q\left(\theta_{R,\varepsilon_R}\right)}=\beta\left[f_R\times\varepsilon_R\times P_R-\frac{1}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[f_R\times\varepsilon_R\times P_R+\theta_{R,\varepsilon_R}\kappa_{R,\varepsilon_R}\right]+\left(1-\delta_R\right)\frac{\kappa_{R,\varepsilon_R}}{q\left(\theta_{R,\varepsilon_R}\right)}\right]
$$

and once we add the assumption that hiring cost if proportional to productivity we get

$$
\begin{aligned}
\frac{\kappa_0}{q\left(\theta_{R,\varepsilon_R}\right)} &= \beta\left[1-\frac{1}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]+\left(1-\delta_R\right)\frac{\kappa_0}{q\left(\theta_{R,\varepsilon_R}\right)}\right] \\
\frac{\kappa_0}{q\left(\theta_{R,\varepsilon_R}\right)}\left(1-\beta\left(1-\delta\right)\right) &= \beta\frac{\frac{1-\tau}{\tau}\frac{1}{1-\sigma}-\theta_{R,\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}} \\
\kappa_0\left[\frac{1-\beta\left(1-\delta\right)}{q\left(\theta_{R,\varepsilon_R}\right)}+\beta\frac{\theta_{R,\varepsilon_R}}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\right] &= \beta\frac{\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}} \\
\kappa_0 &= \frac{\beta\frac{\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}}{\frac{1-\beta(1-\delta)}{q\left(\theta_{R,\varepsilon_R}\right)}+\beta\frac{\theta_{R,\varepsilon_R}}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}}
\end{aligned}
$$

9.2. Productivity cutoffs

Denote the value of staying out of the labor force by $V_{o,\varepsilon}$, a constant number in steady state.

The value of employment in occupation R with idiosyncratic productivity ε_R is

$$
\begin{aligned}
V_{e,R,\varepsilon} &= \frac{\left(\omega_{R,\varepsilon_R}\left(1-T_{e,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}+\beta\left(1-\delta_R\right)V_{e,R,\varepsilon}+\beta\delta_R V_{u,R,\varepsilon} \\
V_{e,R,\varepsilon} &= \frac{1}{1-\beta\left(1-\delta_R\right)}\left[\frac{\left(\frac{f_R\times\varepsilon_R\times P_R}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]\left(1-T_{e,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}\right]+\frac{\beta\delta_R}{1-\beta\left(1-\delta_R\right)}V_{u,R,\varepsilon}
\end{aligned}
$$

where we substituted the explicit wage function under the assumption of proportional hiring costs.

The value of unemployment in occupation R with idiosyncratic productivity ε_R is

$$
\begin{aligned}
V_{u,R,\varepsilon} &= \frac{\left(b_{R,\varepsilon_R}\omega_{R,\varepsilon_R}\left(1-T_{u,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}+\beta\left(1-\mu\left(\theta_{R,\varepsilon_R}\right)\right)V_{u,R,\varepsilon}+\beta\mu\left(\theta_{R,\varepsilon_R}\right)V_{e,R,\varepsilon} \\
V_{u,R,\varepsilon}\left(1-\beta\right) &= \left[\frac{\left(b_{R,\varepsilon_R}\frac{f_R\times\varepsilon_R\times P_R}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]\left(1-T_{u,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}\right]+\beta\mu\left(\theta_{R,\varepsilon_R}\right)\left[V_{e,R,\varepsilon}-V_{u,R,\varepsilon}\right]
\end{aligned}
$$

Note that the first order condition of the bargaining problem implies that

$$
V_{e,R,\varepsilon}-V_{u,R,\varepsilon}=\xi\frac{\tau}{1-\tau}\frac{1}{1-T_\pi}\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}
$$

and the first order condition with respect to vacancies implies that

$$\frac{\partial J\left(x_{R,\varepsilon_R},\Lambda\right)}{\partial x_{R,\varepsilon_R}}=\frac{\left(1-T_\pi\right)\kappa_0 P_R f_R \varepsilon_R}{\beta q\left(\theta_{R,\varepsilon_R}\right)}$$

Substituting, we have

$$V_{u,R,\varepsilon}\left(1-\beta\right)=\left[\frac{\left(b_{R,\varepsilon_R}\frac{f_R\times\varepsilon_R\times P_R}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]\left(1-T_{u,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}\right]$$
$$+\theta_{R,\varepsilon_R}\xi\frac{\tau}{1-\tau}\kappa_0 P_R f_R \varepsilon_R$$

Now we can substitute for ξ, taking into account the CRRA assumption

$$\xi=U'\left(C_{e,R,\varepsilon}\right)\left(1-T_{e,R,\varepsilon}\right)-U'\left(C_{u,R,\varepsilon}\right)\left(1-T_{u,R,\varepsilon}\right)b_{R,\varepsilon}$$
$$=\left(\omega_{R,\varepsilon_R}\right)^{-\sigma}\left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}\right]$$
$$=\left(\frac{f_R\times\varepsilon_R\times P_R}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]\right)^{-\sigma}\left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}\right]$$

Therefore

$$V_{u,R,\varepsilon}\left(1-\beta\right)=\left[\frac{\left(b_{R,\varepsilon_R}\frac{f_R\times\varepsilon_R\times P_R}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]\left(1-T_{u,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}\right]$$
$$+\left(\frac{f_R\times\varepsilon_R\times P_R}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left[1+\theta_{R,\varepsilon_R}\kappa_0\right]\right)^{-\sigma}\left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}\right]\theta_{R,\varepsilon_R}\frac{\tau}{1-\tau}\kappa_0 P_R f_R \varepsilon_R$$
$$=\left(f_R\times P_R\times\varepsilon_R\right)^{1-\sigma}\times\left[\begin{array}{c}\frac{\left(b_{R,\varepsilon_R}\frac{1+\theta_{R,\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\left(1-T_{u,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma}+\\ \left(\frac{1+\theta_{R,\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\right)^{-\sigma}\left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma}-\left(1-T_{u,R,\varepsilon}\right)^{1-\sigma}b_{R,\varepsilon}^{1-\sigma}\right]\theta_{R,\varepsilon_R}\frac{\tau}{1-\tau}\kappa_0\end{array}\right]$$

or

$$V_{u,R,\varepsilon} = \frac{\left(f_R \times P_R \times \varepsilon_R\right)^{1-\sigma}}{1-\beta} \times \left[\begin{array}{l} \frac{\left(b_{R,\varepsilon_R} \frac{1+\theta_{R,\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}} \left(1-T_{u,R,\varepsilon_R}\right)\right)^{1-\sigma}}{1-\sigma} + \\ \left(\frac{1+\theta_{R,\varepsilon_R}\kappa_0}{1+\frac{1-\tau}{\tau}\frac{1}{1-\sigma}}\right)^{-\sigma} \left[\left(1-T_{e,R,\varepsilon}\right)^{1-\sigma} - \left(1-T_{u,R,\varepsilon}\right)^{1-\sigma} b_{R,\varepsilon}^{1-\sigma}\right] \theta_{R,\varepsilon_R} \frac{\tau}{1-\tau}\kappa_0 \end{array} \right]$$

Note that the term in brackets is constant in steady state because it is a combination of exogenous parameters and the tightness ratio, which we have shown to be independent of the productivity parameters. Defining the term in brackets by $\daleth_R$ and the analogue for NRM by $\daleth_{NRM}$ we can express the values of unemployment in both occupations as

$$V_{u,R,\varepsilon} = \frac{\left(f_R \times P_R \times \varepsilon_R\right)^{1-\sigma}}{1-\beta} \times \daleth_R$$

$$V_{u,R,\varepsilon} = \frac{\left(f_{NRM} \times P_{NRM} \times \varepsilon_{NRM}\right)^{1-\sigma}}{1-\beta} \times \daleth_{NRM}$$

10. Appendix: Alternative Calibration

[Missing]

11. Appendix: Policy Experiments: Funding with Labor Taxes

[Missing]